King Fahd University of Petroleum & Minerals

Department of Mathematics and Statistics

Math 201: Second Major Exam, Summer 133 (120 minutes)

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Name: **Student ID:** **Section:** **Serial:**

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Solve all problems. Show **full details** of your solution.

Question		Grade
1		/15
2		/10
3		/15
4		/10
5		/10
6		/10
7		/15
8		/15
TOTAL		

Q1. Consider the two lines

$$L_1 \ : \ \ x = t, \ y = 2 - t, z = 1 + t, \ t \in (-\infty, \infty);$$
$$L_2 \ : \ \ x = 2 + 2s, \ y = 4 + s, \ z = 7 + 5s, \ s \in (-\infty, \infty).$$

(a) (5 points) Find the point of intersection of the two lines.

(b) (10 points) Find the equation of the plane determined by these two lines.

Q2. Consider the surface

$$x = 9 - y^2 - z^2.$$

(a) (6 points) Find the intersection of this surface with the $xy-$plane, the xz-plane and the yz-plane. Identify this surface (*i.e.* name it).

(b) (4 points) Sketch the given surface (labeling the important points and cross sections).

Q3. Consider the function

$$f(x, y) = \cos^{-1}(y - x^2).$$

(a) (6 points) Find the domain of the given function.

(b) (6 points) Sketch the domain of the given function.

(c) (3 points) Find the range of the given function.

Q4. (10 Points) Determine whether each of the following limits exists or not and find its value if it exists (show full details):

a) $\lim\limits_{(x,y)\to(0,0)} x\sin(\frac{1}{y})$.

b) $\lim\limits_{(x,y)\to(1,1)} \frac{xy^2-1}{y-1}$.

Q5. (10 points) Let

$$w = \tan\left(\frac{x}{y}\right) e^{yz},\ x = \ln(s+t),\ y = \cos^{-1}\left(\frac{s}{t}\right),\ z = \sqrt{t-s}.$$

(a) Find $\frac{\partial w}{\partial s}$ at $(s,t) = (0,1)$.

(b) Find $\frac{\partial w}{\partial t}$ at $(s,t) = (0,1)$.

Q6. (10 points) Let

$$xe^y + ye^z + 2\ln x + \ln 3 - 1 = 0$$

(a) Find $\frac{\partial z}{\partial y}$ at $(x, y, z) = (1, \ln 2, \ln 3)$.

(b) Find $\frac{\partial x}{\partial y}$ at $(x, y, z) = (1, \ln 2, \ln 3)$.

Q7. Consider the level surface

$$F(x, y, z) = e^{xy} + \cos(xz) - \tan^{-1}(yz) + \frac{\pi}{4} - 2 = 0.$$

(a) (9 points) Find the equation of the tangent plane to the given level surface at $(0, 1, 1)$.

(b) (6 points) Find the equation of the normal line to the given level surface at $(0, 1, 1)$.

Q8. (a) (7 points) Find the linearization of the following function at $(\frac{\pi}{4}, 1, 1)$:

$$f(x, y, z) = \frac{\sin(xy)}{z}.$$

(b) **(8 points)** Find the directional derivative of

$$f(x, y, z) = e^{xy} + \cos(yz) + \ln(xz)$$

at $P_0(1, 0, 1)$ in the direction of $\overrightarrow{v} =< 2, -2, 1 >$.

GOOD LUCK