King Fahd University of Petroleum & Minerals

Department of Mathematics and Statistics

Math 201: First Major Exam, Summer 133 (120 minutes)

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Name: Student ID: Serial Number:

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Solve all problems. Show **full details** of your solution.

Question		Grade
1		/15
2		/10
3		/15
4		/10
5		/10
6		/10
7		/15
8		/15
TOTAL		

Q1. Let $\mathbf{C}$ be the curve defined by the parametric equations

$$x = 2 + 3\cos(t), \; y = 1 + 3\sin(t), \; 0 \le t \le \pi.$$

(a) (5 points) Eliminate t to find the corresponding cartesian equation for $\mathbf{C}$.

(b) (5 points) Sketch the graph of $\mathbf{C}$ and indicate on it the direction in which $\mathbf{C}$ is traced.

(c) (5 points) Find the area of the surface generated by revolving $\mathbf{C}$ about the x-axis.

Q2. Consider the curve $\mathbf{C}$, whose equation in polar coordinates is given by

$$r = 2\sqrt{2}\sin\left(\theta - \frac{\pi}{4}\right)$$

(a) (6 points) Find the corresponding cartesian equation of $\mathbf{C}$.

(b) (4 points) Sketch the curve of $\mathbf{C}$ in the xy-plane.

Q3. Consider the curve **C** whose equation in polar coordinates is given by

$$r^2 = 4\sin(2\theta).$$

(a) (5 points) Find the symmetries in the graph of **C** (if any).

(b) (5 points) Sketch the graph of **C** in the xy-plane.

(c) (5 points) Find the area of the region enclosed by **C**.

Q4. (10 Points) Find the length of the curve whose equation in polar coordinates is given by

$$r = 2\sec^2\left(\frac{\theta}{2}\right), \ 0 \le \theta \le \frac{\pi}{2}.$$

Q5. (a) (5 points) Give a **complete** geometric description for the locus of points (in the space) satisfying the pair of equations

$$x^2 + y^2 + z^2 = 10, \ y = -1$$

(b) (5 points) Find the set of points in the space equidistant (*i.e.* having the same distance) from the origin and the point $(2, 0, 0)$.

Q6. (a) (5 points) Find the vector $\vec{v}$ whose length is 2 making the angle $\frac{7\pi}{4}$ with the positive x-axis.

(b) (5 points) Find a vector $\vec{v}$ whose length is 100 and whose direction is opposite to that of $\vec{u} =< \frac{1}{2}, \frac{1}{2}, -\frac{1}{2} >$.

Q7. (a) (9 points) Find the measures of the angles of the triangle whose vertices are $A(-1, 0)$, $B(2, 1)$ and $C(1, -2)$.

(b) (6 points) Is $\vec{v} = \,<2, -3>$ perpendicular to the line L whose equation is $2x - 3y = 6$? (Justify your answer)

Q8. **(a) (8 points)** Find the area of the triangle whose vertices are $A(1, -1, 1)$, $B(0, 1, 1)$ and $C(1, 0, -1)$.

(b) **(7 points)** Determine whether the points $A(1, 3, 2)$, $B(3, -1, 6)$, $C(5, 2, 0)$ and $D(3, 6, -4)$ lie in the same plane or not. (Justify your answer).

GOOD LUCK