KING FAHD UNIVERSITY OF PETROLEUM AND MINERALS

DEPARTMENT OF MATHEMATICAL SCIENCES
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STAT 591: Special Topics in StatisticsI

Final Exam, Semester- 132, Year 2014
Time: 7:00 pm to 10:00 pm, Tuesday, 6th May, 2014

Student Name: ……..ID# ……..……

Q1. (10 points)

Let X be a continuous random variable (quality characteristic of interest in a process) with mean μ and standard deviation σ (i.e. variance σ^2) and is normally distributed. Let $x_1, x_2, ..., x_n$ be a random sample of size n form the process.

Using this information to answer the following:

i) find the probability limits of $\bar{X}$ chart at n=6 and ARL_0 =250

ii) using the probability limits in (i) compute ARL_1 values for $\delta = 0.5, 1, 2, 4$ where δ is defined as: $\mu_1 = \mu_0 + \delta\sigma_0$

Q.No.2: (10 points)

For a process, we want to apply S-chart, and the in-control sigma is known to be $\sigma_0 = 22$. In future, the samples of size $n = 20$ are to be selected for monitoring.

(i) Derive the 3-sigma limits of S-chart.

(ii) Find the values of $L_{\frac{\alpha}{2}}$ and $L_{1-\frac{\alpha}{2}}$ for which the probability limits become same as the 3-sigma limits.

Q3: (10 points)

A process a normally distributed with known mean μ_0 and known variance σ_0^2 and random samples of size *n=5* are drawn from the process to monitor the stability of process parameters. The process variance remains stable at σ_0^2 and mean shifts to new level $\mu_1 = \mu_0 + \delta\sigma_0$. The following table provides the ARL values at different shifts in terms of δ.

δ	CUSUM				EWMA				Shewhart
	k				λ				
	0.50	**1.00**	**2.00**	**3.00**	**0.25**	**0.50**	**0.75**	**1.00**	
0.00	370.37	370.37	370.37	370.37	370.37	370.37	370.37	370.37	370.37
0.25	28.29	54.62	121.75		32.82	57.51	90.77		
0.50	8.35	10.36	27.05		8.37	11.67	19.06		
0.75	4.78	4.41	8.15		4.39	4.76	6.45		
1.00	3.39	2.75	3.53		3.01	2.84	3.19		
1.50	2.24	1.63	1.48		1.97	1.61	1.49		
2.00	1.80	1.17	1.07		1.47	1.15	1.09		
2.50	1.38	1.02	1.01		1.11	1.02	1.01		
3.00	1.08	1.00	1.00		1.01	1.00	1.00		
4.00									
5.00									
10.00									

i) Fill in the missing cells of the table (show computations of few selective cells)

ii) Discuss the effects of *k and* λ in CUSUM and EWMA respectively.

iii) Provide a comparative discussion of the tabular results of Shewhart, CUSUM and EWMA.

Q4: (10 points)

Show that if $\lambda = 2/(w+1)$ for the EWMA control chart then this control chart is equivalent to a w-period moving average control chart in the sense that the control limits are identical in the steady state.

(You may see the details of moving average control chart on page 428, section 9.3 of Montgomery Book available in you webct)

Q5: (10 points)

a). Define a control chart and briefly discuss it with reference to phase I and phase II monitoring.

b). Consider a single sampling plan with n=80 and c=3. Complete the following table for OC curve using different lot qualities:

P	**P_a**
0.00	
0.01	
0.05	
0.10	
0.25	
0.50	
0.80	
0.95	
0.99	

The probability expression for $\mathbf{P_a}$ =……

If AQL (p_1) = 0.01 then PR = …….

If RQL (p_2) = 0.10 then CR = …….

Q6. (20 points)

Refer to datasets called carbon 1 and carbon 2, and do the following using R language.

i. Provide scatterplot of Principal component scores with 99% confidence ellipse in phase I for carbon 1 and in phase II for carbon 2.

ii. Construct Hotelling T^2 control chart of the principal component scores in phase I for carbon 1 and perform phase II analysis for carbon 2.

iii. Construct a Pareto chart of the principal components summary using both the carbon 1 and carbon 2 datasets separately.

iv. Construct Generalized variance control chart in phase I for carbon 1 and perform phase II analysis for carbon 2.

(Please prepare the solution to this question in soft form as a word file. It should include the commands and the outputs of your results. Send me email of this solution before leaving the exam hall).

Q7. (30 points)-----(Take Home)

a). Refer to datasets called carbon 1 and carbon 2, and do the following using R language:.

v. Provide data description for both the datasets.

vi. Construct 3D scatterplot with the 99% confidence region and comment on the output for both the datasets.

vii. Provide Matrix of scatterplot for both the datasets.

viii. Construct Hotelling T^2 control chart in phase I for carbon 1 and perform phase II analysis for carbon 2.

ix. Construct MEWMA control chart with λ =0.1 in phase I for carbon 1 and perform phase II analysis for carbon 2.

x. Construct MCUSUM control chart according to Crosier (1988) in phase I for carbon 1 and perform phase II analysis for carbon 2.

xi. MCUSUM control chart according to Pignatiello and Runger (1990) in phase I for carbon 1 and perform phase II analysis for carbon 2.

xii. Provide scatterplot of Principal component scores with 99% confidence ellipse in phase I for carbon 1 and in phase II for carbon 2.

xiii. Construct Hotelling T^2 control chart of the principal component scores in phase I for carbon 1 and perform phase II analysis for carbon 2.

xiv. Construct Generalized variance control chart in phase I for carbon 1 and perform phase II analysis for carbon 2.

xv. Construct a Pareto chart of the principal components summary using both the carbon 1 and carbon 2 datasets separately.

b). Eliminate the rational subgrouping for the two datasets and obtain 2D arrays for them. Assuming the process specifications given by:

LSL = [0.60, 0.30, 49.00], USL = [1.40, 1.70, 51.00], Target = [1.00, 1.00, 50.00]

Do the following using R language:

i. Compare the Pan and Lee (2010) NMCpm index with the first component of the Shahriari et al. (1995) vector using alpha = 0.0001.

ii. Contrast both indices but setting the midpoint between specifications as target.