KING FAHD UNIVERSITY OF PETROLEUM & MINERALS
DEPARTMENT OF MATHEMATICS AND STATISTICS
Term 132

STAT 416 Stochastic Processes for Actuaries

Final Exam   Monday May 19, 2014

Name: ____________________________ ID #: ________________

Important Note:

- Show all your work including formulas, intermediate steps and final answer

Question No	Full Marks	Marks Obtained
1	7	
2	8	
3	5	
4	7	
5	13	
6	10	
7	10	
Total	60	

Q1:

a. A computer system has exponential failure time density function

$$f(t) = \frac{1}{9000} e^{-\frac{t}{9000}}, \qquad t > 0$$

What is the probability that the system will fail after the warranty (six months or 4380 hours) and before the end of the first year (one year or 8760 hours)? (2 points)

b. Suppose that a particular electronic system consists of four components (A, B, C, and D) connected in parallel. The time to failure for each component is exponentially distributed with mean times between failures of 200, 200, 300, and 600 hours, respectively. Determine the system reliability for 1000 hours, assuming that each component operates independently and that each begins operation when the system begins operation. (5 points)

Q2: Machines in a factory break down at an exponential rate of six per hour. There is a single repairman who fixes machines. The service time to fix a machine is normally distributed with mean 9 minutes and standard deviation of 2 minutes.

1. Specify the type of the system. (4 points)

2. What is the expected number of machines waiting in line to be fixed? (2 points)

3. What is the average time a machine spends to work again? (2 points)

Q3: Three out of four trucks on the road are followed by a car, while only one out of every five cars is followed by a truck.

1. Show how this system can be analyzed by using Markov chain, how many states are needed? (2 points)

2. In the long run, what fraction of the vehicles on the road are trucks? (3 points)

Q4: the current price of a stock is \$100. The price follows a geometric Brownian motion with drift rate 12% per year and variance 10% per year.

1. Calculate the expected value of the stock price after two years from now. (2 points)

2. What is the probability that the price will be at least \$210 after one and half year. (5 points)

Q5: Let $\mathrm{B(t)}, \mathrm{t} \geq 0$ is a standard Brownian motion process.

1. State the assumptions of this Stochastic process (2 points)

2. Find $E\big(B(2)B(3)\big) =$ (2 points)

3. Find $Var\left(\int_0^1 t^2\, dB(\mathrm{t})\right) =$ (2 points)

4. Let $X(\mathrm{t}) = \int_0^t \mathrm{B}^2(\mathrm{s})\, ds$, find the mean for $X(\mathrm{t})$ (2 points)

5. Let $Y(\mathrm{t}) = tB\left(\frac{1}{t}\right) - B(1)$. Find the mean for $\mathrm{Y(t)}$ (2 points)

6. Let $Z(t) = \sigma B(\mathrm{t}) + \mu t$. Show that the process $\mathrm{A(t)}$ is a geometric Brownian motion where $\mathrm{A(t)} = e^{Z(t)-\mu t-\frac{1}{2}\sigma^2 t}$ (3 points)

Q6: A discrete random variable X has a probability mass function satisfying the recursive equation

$$p_x = \frac{2}{x} p_{x-1} \quad for\ x = 1, 2, 3, \ldots$$

1. Explain a procedure to generate values from this distribution and satisfy your procedure. (6 points)

2. Using the following 4 random numbers (U). (4 points)

   0.6 0.25 0.02 0.93

   Generate 4 values from this distribution

Q7: Suppose that you are an actuary of an insurance company in charge to assessing the probability of a life insurance portfolio through simulation. As a first step, you need to estimate future lifetimes of policyholders in the portfolio. After some preliminary investigation of the portfolio's historical data, you are convinced that future lifetimes can be modeled with a Weibull distribution with density function

$$f(x) = 4xe^{-2x^2} \quad for\ x > 0$$

1. Explain whether it is possible to use the ***inverse transformation*** method to simulate from this distribution. (5 points)

2. Describe a procedure that applies the ***rejection method*** to obtain simulated values from this distribution based on the half normal distribution with density

$$g(x) = \sqrt{\frac{2}{\pi}} e^{-\frac{x^2}{2}} \quad for\ x > 0$$

(**Hint**) You may assume that you have a procedure for simulating from a standard normal $Z \sim Normal\ (0, 1)$ (5 points)