KING FAHD UNIVERSITY OF PETROLEUM & MINERALS
DEPARTMENT OF MATHEMATICS AND STATISTICS
Term 132

STAT 416 Stochastic Processes for Actuaries

Third Exam Monday May 5, 2014

Name: ________________________________ ID #: ____________________

Important Note:

- Show all your work including formulas, intermediate steps and final answer

Question No	Full Marks	Marks Obtained
1	10	
2	10	
3	10	
4	10	
Total	40	

Q1: Patients arrive independently to the emergency room in according to a Poisson process with rate one every half hour. The time spent with the emergency doctor is exponentially distributed with average service time equal to 20 minutes to treat each patent

1. Specify the type of the system. (2 points)

2. Does this queueing system reach steady state? Why? (1 point)

3. Find the probability that there are at least two patients waiting to see the doctor. (2 points)

4. Find the expected number of patients in the emergency room. (1 point)

5. Find the expected number of patients waiting the doctor. (1 point)

6. Find the expected time for a patient in the emergency room. (1 point)

7. Find the probability that a patient waits more than 30 minutes in the emergency room. (2 points)

Q2: Consider a bank with two tellers in which customers arrive with rate λ per hour arrive at the bank and wait in a single line for an idle teller. The average time it takes to serve a customer is $\frac{1}{\mu}$ hours. The bank opens daily at 9:00 am and closes at 4:00 pm. Assume that inter-arrival times and service times are exponential

1. Construct the rate diagram for this system (2 points)

Set up the balance equations and solve for the steady – state probabilities of the number of

3. In the long – run, what proportion of time is both tellers busy? (3 points)

Q3: Assume that a part of a domestic wastewater treatment station constitutes a system, with 6 components and structure function given by:

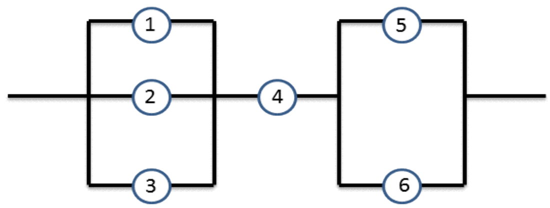
$$\phi(\mathbf{x}) = 1 - (1 - x_1x_2x_3x_6)(1 - x_1x_2x_5x_6)(1 - x_1x_4x_5x_6)(1 - x_1x_3x_4x_6)$$

$$= \big(1 - (1 - x_1)\big)\big(1 - (1 - x_2)(1 - x_4)\big)\big(1 - (1 - x_3)(1 - x_5)\big)\big(1 - (1 - x_6)\big)$$

1. Identify the minimal path sets and minimal cut sets, and draw a reliability block diagram as close as possible of the system (5 points)

2. Suppose that each of those 6 components are independent and have reliability $p_j = p = 0.95,\ j = 1,2,\dots,6$. Calculate the upper and the lower bounds of the reliability function. (5 points)

Q4: The figure below is a reliability block diagram for a part of a computer system:



Assume the durations in 10^3 hours of the 6 components are independent random variable with common Uniformly distributed over the interval $(0, 1), i = 1, 2, \ldots, 6$. For this part of the computer system

1. Obtain the structure function. (2 points)

2. Obtain the reliability. (3 points)

3. Find the component lifetime distribution $F_i(t)$ (3 points)

4. Find the expected system life (2 points)