KING FAHD UNIVERSITY OF PETROLEUM & MINERALS
DEPARTMENT OF MATHEMATICS AND STATISTICS
Term 132

STAT 416 Stochastic Processes for Actuaries

Second Exam Monday March 10, 2014

Name: ____________________________ ID #: ________________

Important Note:

- Show all your work including formulas, intermediate steps and final answer

Question No	Full Marks	Marks Obtained
1	10	
2	6	
3	7	
4	5	
5	8	
6	10	
Total	45	

Q1:

a. Consider a Markov process with states $0, 1, 2$ and which the following transition ratio matrix Q

$$Q = \begin{bmatrix} -\lambda & \lambda & 0 \\ \mu & -\lambda - \mu & \lambda \\ \mu & 0 & -\mu \end{bmatrix}$$

Where $\lambda > 0,\ \mu > 0$

Derive the parameters v_i and P_{ij} for this Markov process. (5 points)

b. If the mean – value function of a renewal process $\{N(t) \geq 0\}$ is given by $m(t) = 2t,\ \ t \geq 0$,

1. What is the distribution of $N(t)$? (2 points)

2. Find $P(N(10) > 1)$ (3 points)

c. Consider a birth – and – death process with birth rates $\lambda_i = (i+1)\lambda,\ \ i \geq 0$ and death rates $\mu_i = i\mu,\ \ i \geq 0$, determine the expected time to go from state 2 to state 5 (5 points)

Q2: For the two state Markov chain with transition matrix

$$Q = \begin{bmatrix} -\mu q & \mu q \\ \mu p & -\mu p \end{bmatrix}$$

Where $\mu > 0, \quad p + q = 1$

Show that the continuous Markov chain subordinate to the Poisson process of rate μ has the transition matrix

$$P(t) = \begin{bmatrix} p + qe^{-\mu t} & q - qe^{-\mu t} \\ p - pe^{-\mu t} & q + pe^{-\mu t} \end{bmatrix}$$

(10 points)

Q3: A small barbershop, operated by a single barber, has room for at most two customers. Potential customers arrive at a Poisson rate of three per hour, and successive service time are independent exponential random variable with mean $\frac{1}{4}$ hour.

1. What is the average number of customers in the shop? (2 points)

2. What is the proportion of potential customers that inter the shop? (2 points)

3. If the barber could work twice as fast, how much more business would he do? (2 points)

Q4: Consider a renewal process $\{N(t) \geq 0\}$ having $a\, uniform(0,1) \;\; for \;\; t < 1$ inter-arrival distribution. Find the renewal function $m(t)$ (5 points)

Q5: a teller continually process unending of job. The time that takes the teller to process a job is a gamma random variable with parameters $n = 4, \lambda = 2$. Approximate that the probability that the teller can process at most 45 jobs by time $= 100$. (5 points)

Q6: Assume that the rate an insurance policyholders alternates between r_1 and r_0 per a month. A new policyholder is initially charged at a rate of r_1 per month. The rate becomes from r_0 if no claims are made for s months. The rate remains at r_0 until a claim is made. Then, initial rate r_1 becomes effective. Suppose that the contract lasts forever and makes claims at times chosen according to a Poisson process with rate λ

Find

1. The proportion of time that the policyholder pays at rate $r_i, i = 0,1$ (5 points)
2. The long – run average amount paid per month. (2 points)