KING FAHD UNIVERSITY OF PETROLEUM & MINERALS
DEPARTMENT OF MATHEMATICS AND STATISTICS
Term 132

STAT 416 Stochastic Processes for Actuaries

First Exam Monday March 10, 2014

Name: _____ ID #: _____

Important Note:

- Show all your work including formulas, intermediate steps and final answer

Question No	Full Marks	Marks Obtained
1	10	
2	6	
3	7	
4	5	
5	8	
6	10	
Total	45	

Q1: Specify the states of the following Markov chains, and determine whether they are transient or recurrent (hint: draw the transient diagram) (10 points)

1. $$P = \begin{bmatrix} 0 & 0.5 & 0.5 \\ 0.5 & 0 & 0.5 \\ 0.5 & 0.5 & 0 \end{bmatrix}$$

2. $$P = \begin{bmatrix} 0.5 & 0 & 0.5 & 0 & 0 \\ 0.25 & 0.5 & 0.25 & 0 & 0 \\ 0.5 & 0 & 0.5 & 0 & 0 \\ 0 & 0 & 0 & 0.5 & 0.5 \\ 0 & 0 & 0 & 0.5 & 0.5 \end{bmatrix}$$

Q2: A Markov chain with three states $P = \begin{bmatrix} \frac{1}{2} & \frac{1}{3} & \frac{1}{6} \\ 0 & \frac{1}{3} & \frac{2}{3} \\ \frac{1}{2} & 0 & \frac{1}{2} \end{bmatrix}$

If $P(X_0 = 0) = P(X_0 = 1) = \frac{1}{3}$ and $P^2 = \frac{1}{18}\begin{bmatrix} 6 & 5 & 7 \\ 6 & 2 & 10 \\ 9 & 3 & 6 \end{bmatrix}$, find $E(X_2)$ (5 points)

Q3: Suppose that students arrive to building 6 at a Poisson rate 1.5 per minutes

1. What is the probability that the elapsed time between the 100^{th} arrival and the next arrival exceeds 2 minutes? (2 points)

2. What is the expected time until the 100^{th} student arrives? (2 points)

3. What is the probability that the 100^{th} arrival will arrive after 75 minutes? You may approximate this probability with a Normal distribution. (3 points)

Q4: Assume that customers in a department store arrive at a Poisson rate that increases linearly for 6 per hour at 1 PM to 9 per hour at 2 PM. Calculate the probability that exactly 2 customers arrive between 1 PM and 2 PM. (5 points)

Q6: An insurance company has two types of policy: policy A and policy B. the total claims from the company arrive according to a Poisson process at rate 9 per day. A randomly selected claim has $\frac{1}{3}$ chance that it is of policy A.

1. Calculate the probability that claims from polity B will be fewer than 3 on a given day (5 points)

2. A randomly selected claim from policy A has a $\frac{2}{3}$ probability of being over \$10000 while a randomly selected claim from policy B has a probability $\frac{2}{9}$ of being over \$10000. Determine the expected number of claims over \$10000 on a given day. (3 points)

Q6: An insurance company charges customers annual premiums based on their accident history in the following fashion:

- No accident in last 2 years: $250 annual premium
- Accidents in each of last 2 years: $800 annual premium
- Accident in only 1 of last 2 years: $400 annual premium

If a customer had an accident last year then they have a 10% chance of having one this year; If they had no accident last year then they have a 3% chance of having one this year.

1. Find the transition probability matrix for this process (3 points)
2. Find the limiting probabilities (5 points)
3. Find the long – run average annual premium paid by the customers. (2 points)