1) Let X and Y have the joint pdf

$$f(x,y) = \begin{cases} 2 & for\ 0 < x < y < 1 \\ 0 & otherwise \end{cases}$$

   a) Find the marginal density of *X*.
   b) Find the marginal density of *Y*.
   c) Find the conditional density of *X* given $Y = y$
   d) Find the expected value of *X* given $Y = y$
   e) Find the variance of *X* given $Y = y$
   f) Find $P\left(X < \frac{1}{2}\right)$
   g) Find $P\left(X < \frac{1}{2} \middle| Y = \frac{3}{4}\right)$
   h) Find the correlation coefficient ρ of *X* and *Y*.
   i) Find the joint moment generating function of *X* and *Y*, and use it to find the moment generating function of *X*.

2) Suppose that the random variables $X_i$ are independent Poisson random variables with means $\lambda_i$ , $i=1, 2, 3$, respectively. Let $Y = X_1 + X_2$ and $W = X_2 + X_3$. Find the joint probability mass function $P(Y = i, W = j)$

3) If $X = ln\ Y$ has a normal distribution with mean μ and variance $\sigma^2$, *Y* is said to have a lognormal distribution. Find the pdf of *Y*.

4) A fair die is rolled until all 6 sides have appeared at least once.
   a) Find the expected number of rolls until all 6 sides appear.
   b) Use this result to find the expected number of times that outcome 1 appears.

5) The number of automobiles sold weekly is a random variable with expected value 16. Give an upper bound to the probability that next week's sales exceed 18.

6) Suppose that the number of units produced daily at factory A is a random variable with mean 20 and standard deviation 3, and the number of units produced daily at factory B is a random variable with mean 18 and standard deviation 6. Assuming independence, find an upper bound for the probability that more units are produced at factory B than at factory A.

7) If X is a positive random variable with mean μ. Show that $P(X > 2\mu) \leq \frac{1}{2}$

8) If X is a positive random variable, what is the relationship between $E\left(1/X\right)$ and $1/E(X)$

9) Walid will take two books with him on a trip. Suppose that the probability that he will like book 1 is 0.6, the probability that he will like book 2 is 0.5, and the probability that he will like both books is 0.4. What is the conditional probability that he will like book 2 given that he did not like book 1.

10) Six balls are to be randomly chosen from an urn containing 8 red, 10 green and 12 blue balls.
   a) What is the probability at least one red ball is chosen?
   b) Given that no red balls are chosen, what is the probability that there are exactly 2 green balls among the 6 chosen?