KING FAHD UNIVERSITY OF PETROLEUM & MINERALS
DEPARTMENT OF MATHEMATICS AND STATISTICS
Term 132

STAT 301 Probability Theory

Second Major Exam Monday April 28, 2014

Name: ______________________________ ID #: ________________

1) Find *E(X)* if *X* have the following density function *(2 pts.)*

$$f(x)=\begin{cases} c\left(1-x^2\right) & -1<x<1 \\ 0 & otherwise \end{cases}$$

2) Let *X* be a Poisson random variable with parameter λ. The truncated Poisson random variable *Y* is defined by $P\left(Y=i\right)=P\left(X=i \mid X>0\right)$. Find *E(Y).* *(4 pts.)*

3) An insurance salesman scheduled two appointments to sell policies. His first appointment will lead to a sale with probability 0.3 and his second will lead independently to a sale with probability 0.6. Any sale made is equally likely to be either for life insurance, which costs SR 1000 or house insurance which costs SR 500. Determine the probability mass function of X, the total value of all sales. *(5 pts.)*

4) Let X be a binomial random variable with parameters n and p. Find $E\left[\frac{1}{X+1}\right]$.

(4 pts.)

5) You arrive at a bus stop at 10 o'clock, knowing that the bus will arrive at some time uniformly distributed between 10:00 and 10:30.

   a) What is the probability that you will have to wait longer than 10 minutes? *(2 pts.)*

   b) If, at 10:15 the bus has not yet arrived, what is the probability that you will have to wait <u>at least</u> an additional 10 minutes? *(3 pts.)*

6) Let X be a continuous random variable with cumulative distribution function F. Define the random variable Y by $Y = F(X)$. What is the distribution of Y? *(3 pts.)*

7) The joint density of is

$$f(x,y)=\begin{cases} xy & 0<x<1, \quad 0<y<2 \\ 0 & otherwise \end{cases}$$

a) Find the density of *X*. *(2 pts.)*

b) Find the density of *Y*. *(2 pts.)*

c) What can you say about *X* and *Y*? *(1 pt.)*

d) Find the joint distribution function. *(3 pts.)*

e) Find *P[X+Y < 1]*. *(3 pts.)*

8) Let X and Y be independent uniform (0 , 1) random variables.

a) Find the joint density of $U = X$ and $V = X+Y$. *(3 pts.)*

b) Use the result in a) to compute the density of V. *(3 pts.)*