KING FAHD UNIVERSITY OF PETROLEUM & MINERALS
DEPARTMENT OF MATHEMATICS AND STATISTICS
Term 132

STAT 319 Probability Theory

First Major Exam — Monday March 10, 2014

Name: Khalid — ID #: ______________

1) Let M denote the larger of the number of points showing when throwing two dice, or their common value if equal. What is the probability distribution of M? *(4pts.)*

m	1	2	3	4	5	6
$P(M=m)$	$\frac{1}{36}$	$\frac{3}{36}$	$\frac{5}{36}$	$\frac{7}{36}$	$\frac{9}{36}$	$\frac{11}{36}$

2) A newly formed club of ten members decides to select its officers by lottery. From the ten names, four are drawn out one by one. The first one drawn will serve as president, the second as vice president, the third as treasurer and the fourth as secretary.

a) How many different possible outcomes are there? *(2pts.)*

$$10 \cdot 9 \cdot 8 \cdot 7 = 5040$$

b) What is the probability that the only girl in the club will become vice president? *(2pts.)*

$$\frac{9 \cdot 1 \cdot 8 \cdot 7}{5040} = \frac{1}{10}.$$

3) In a food tasting experiment, a subject is asked to rate five brands of coffee which are sold at different prices. The coffee is served to the subject in five cups in random order. Find the probability that

a) The first 3 cups served are, in order, the most expensive, the next most expensive, the third most expensive. *(2pts.)*

$$\frac{1 \cdot 1 \cdot 1 \cdot 2}{5!} = \frac{1}{60}$$

b) The first three cups served are the three most expensive brands, but not necessarily in order. *(2pts.)*

$$\frac{3!\,2!}{5!} = \frac{1}{10}.$$

4) Urn 1 contains 5 white and 7 black balls. Urn 2 contains 4 white and 2 black balls. An urn is selected at random, and a ball is drawn from it. If the drawn ball is white, what is the probability that urn 1 was chosen? *(4pts.)*

$$P(\text{urn 1} \mid \text{Ball is white}) = \frac{P(\text{urn 1 and Ball is white})}{P(\text{Ball is white})}$$

$$= \frac{P(\text{Ball is white} \mid \text{Urn 1})\,P(\text{urn 1})}{\left\{ \begin{array}{l} P(\text{Ball is white} \mid \text{Urn 1})\,P(\text{urn 1}) + \\ P(\text{Ball is white} \mid \text{Urn 2})\,P(\text{urn 2}) \end{array} \right\}}$$

$$= \frac{\frac{1}{2} \cdot \frac{5}{12}}{\frac{1}{2} \cdot \frac{5}{12} + \frac{1}{2} \cdot \frac{4}{6}}$$

$$= \frac{5}{13}.$$

5) An urn containing 6 balls, 4 of which are white. Let E denote the event that exactly 1 of the balls drawn on the first 2 draws is white, and F be the event that the ball drawn on the fourth draw is white. Are E and F independent? Justify your answer. *(4pts.)*

$$P(E) = \frac{6}{15} = \frac{2}{5}; \quad P(F) = \frac{7}{15}$$

$$P(EF) = \frac{4}{15} \neq P(E) \cdot P(F)$$

Hence they are not independent

6) Show that if E and F are independent, then

a) E and F^c are independent. *(2pts.)*

$$E = EF \cup EF^c$$

$$P(E) = P(EF) + P(EF^c)$$

$$P(EF^c) = P(E) - P(EF)$$

$$= P(E) - P(E)P(F)$$

$$= P(E)[1 - P(F)] = P(E) \cdot P(F^c).$$

b) E^c and F^c are independent. *(2pts.)*

From (a) E & F^c are independent

$\Rightarrow F^c$ and E^c are independent

Bonus *(6 points)*

7) Let E and F be two events with positive probability, prove or disprove:

a) $P(E|F) + P(E^c|F) = 1$ *(2pts.)*

$$P(E|F) = \frac{P(EF)}{P(F)}$$

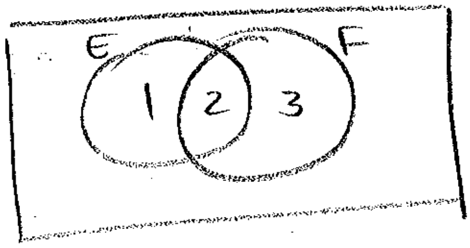
$$P(E^c|F) = \frac{P(E^cF)}{P(F)}$$

$\Rightarrow$

$$\frac{P(EF)}{P(F)} + \frac{P(E^cF)}{P(F)} = \frac{P(EF) + P(E^cF)}{P(F)} = \frac{P(F)}{P(F)} = 1$$

Since $F = EF \cup E^cF$.

b) $P(E|F) + P(E|F^c) = 1$ False *(2pts.)*



$P(E|F) = \frac{1}{2}$

$P(E|F^c) = 1$

8) Show that if E and F are mutually exclusive, they cannot be independent unless........(what?) *(2pts.)*

$\rightarrow$ one of the events has 0 probability.

$P(EF) = 0$

$P(E)P(F) = 0 \Rightarrow$

either $P(E) = 0$

or $P(F) = 0$