King Fahd University of Petroleum & Minerals

Department of Mathematics and Statistics

Math 690: Special Topics in Mathematics

Final Exam, Fall Semester 132 (150 minutes)

Jawad Abuhlail

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Q1. (40 Points) Prove *four* of the following statements:

(1) Any two skeletons of a given category are isomorphic.

(2) Every regular epimorphism is an extremal epimorphism.

(3) A functor $F : \mathcal{A} \longrightarrow \mathcal{B}$ is an equivalence (*i.e.* F is full, faithful and isomorphism-dense) if there exists a functor $G : \mathcal{B} \longrightarrow \mathcal{A}$ such that $G \circ F \simeq \mathrm{id}_{\mathcal{A}}$ and $F \circ G \simeq \mathrm{id}_{\mathcal{B}}$.

(4) Terminal objects are essentially unique.

(5) Regular monomorphisms are pullback stable.

(6) Every category with products and equalizers is complete.

Q2. (15 Points) Give an *example* for (at least) *three* of the following:

(1) A concrete functor from **Set** to **Top**.

(2) A coseparator in **Set**.

(3) A discrete object in **Top**.

(4) A category in which every object is injective.

Q3. (15 Points) Give a *counter example* to (at least) *three* of the following *wrong* statements:

(1) Every bimorphism is an isomorphism.

(2) Every embedding is full.

(3) Any two equivalent categories are isomorphic.

(4) Every object in **Ab** is injective.

Q4. (15 Points) Fill in (at least) *three* gaps to obtain a correct statement:

(1) If $G \circ F$ is full and ...., then G is full.

(2) Every full ...... functor reflects sections.

(3) If f is an extremal monomorphism and ...., then $g \circ f$ is an extremal monomorphism.

(4) If $g \circ f$ is an essential embedding and ....., then g is an essential embedding.

Q5. (15 Points) Indicate whether *each* of the following statements is TRUE or FALSE:

(1) Any two categories with isomorphic skeletons are isomorphic.

(2) The category **Rng** has a zero object.

(3) The category of torsion-free Abelian groups is balanced.

(4) Monomorphisms are pullback stable.

(5) $f : A \longrightarrow B$ is a monomorphism if and only if the following square is a pullback

$$\begin{array}{ccc} A & \xrightarrow{id_A} & A \\ {\scriptstyle id_A}\downarrow & & \downarrow{\scriptstyle f} \\ A & \xrightarrow[f]{} & B \end{array}$$

GOOD LUCK