King Fahd University of Petroleum & Minerals
Department of Math. & Stat.

Math 668 - Final Exam (132)   Time: 2 hours 30 mns

May 25, 2014

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Name: ID #

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Problem 1	/10
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Problem 2	/10
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Problem 3	/10
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Total	/30

Problem # 1. (10 marks) Let Ω be a bounded and smooth domain of $\mathbb{R}^n$. Consider the problem

$$(\mathrm{P}_1) \qquad \begin{cases} u_t(x,t) - \left(\int_\Omega |\nabla u(x,t)|^2 dx\right) \Delta u(x,t) = f(x,t) & \text{in } \Omega \times (0,+\infty) \\ u(x,0) = u_0(x) & \text{in } \Omega \\ u = 0 & \text{on } \partial\Omega \times [0,+\infty) \end{cases}$$

where $u_0 \in H_0^1(\Omega)$ and $f \in L^2(\Omega \times (0,+\infty))$. Show that (P_1) has a unique weak solution $u \in L^2((0,+\infty); H_0^1(\Omega))$.

Problem # 2. (10 marks) In a bounded and smooth domain of $\mathbb{R}^n$, consider the problem

$$(\mathrm{P}_2) \qquad \begin{cases} u_{tt}(x,t) + \Delta^2 u(x,t) = 0 & \text{in } \Omega \times (0,+\infty) \\ u(x,0) = u_0(x), \qquad u_t(x,0) = u_1(x) & \text{in } \Omega \\ \frac{\partial u}{\partial \nu} = u = 0 & \text{on } \partial\Omega \times [0,+\infty) \end{cases}$$

If $u_0 \in H^4(\Omega) \cap H_0^2(\Omega)$ and $u_1 \in H_0^2(\Omega)$, show that (P_2) has a unique solution

$$u \in C\left([0,+\infty), H^4(\Omega) \cap H_0^2(\Omega)\right) \cap C^1\left([0,+\infty), H_0^2(\Omega)\right) \cap C^2\left([0,+\infty), L^2(\Omega)\right)$$

Problem # 3. (10 marks) Given the nonlinear problem

$$(\mathrm{P}_3) \qquad \begin{cases} u_{tt}(x,t) - \Delta u(x,t) + h(u(x,t)) = 0 & \text{in } \Omega \times (0,+\infty) \\ u(x,0) = u_0(x), \qquad u_t(x,0) = u_1(x) & \text{in } \Omega \\ u = 0 & \text{on } \partial\Omega \times [0,+\infty) \end{cases}$$

where Ω is a bounded and smooth domain of $\mathbb{R}^n$, $u_0 \in H_0^1(\Omega)$, $u_1 \in L^2(\Omega)$ and $h : \mathbb{R} \to \mathbb{R}$ is such that

$$|h(s)| \leq \alpha |s| \text{ and } H(s) = \int_0^s h(\xi) d\xi \geq 0, \qquad \alpha > 0.$$

a. Use Galerkin method to establish an existence result for (P_3)

b. If h is increasing, show that the solution is unique.