HW

1. (a) Give an example of functions f and g, continuous in a neighboourhood of zero, so that $f(x) = O(g(x))$ but it is not true that $\lim_{x\to 0} |f(x)|/|g(x)| = L$ for some finite L.

(b) What about the special case where $g(x) = x^\alpha$, $\alpha > 0$?

(c) (Optional) Remind yourself what the *limit superior* means. Is (a) still true if lim is replaced by lim sup?

2. Obtain a three term asymptotic approximation (for $\epsilon \to 0$) of the roots of

(a) $x^2 + 4x - 5 - \epsilon = 0$

(b) $x^2 - (5 + 2\epsilon)x + 6 - \epsilon = 0$

(c) $x^2 + (4 + \epsilon)x + 4 - \epsilon = 0$

(d) $\epsilon x^2 + 2x - 4 = 0$

(e) $x^3 - (3+\epsilon)x + \epsilon - 2 = 0$, in this case only obtain a two term expansion for each of the three roots.

(f) $(x + 1)^3 = \epsilon(x^2 - x + 6)$, again only obtain the first two terms. You should find that all three roots are the same to this order.