King Fahd University of Petroleum and Minerals

Department of Mathematics and Statistics

Math 605
Exam I– 2014–2015 (132)
Wednesday, March 19, 2014

Allowed Time: 120 minutes

Instructor: **Dr. Boubaker Smii**

Name: ______________________________

ID #: ____________________

Section #: ___________ Serial Number: ___________

Instructions:

1. Write clearly and legibly. You may lose points for messy work.
2. **Show all your work**. No points for answers without justification !

Question #	Grade	Maximum Points
1		13
2		07
3		16
4		12
5		12
Total:		60

Exercise 1:

Use Stirling's approximation to show that

$$\frac{\Gamma(x+a)}{\Gamma(x+b)} \sim x^{a-b}, \; x \to +\infty$$

Exercise 2:

Consider the integral

$$\int_0^\infty e^{-xt} e^{-\frac{1}{t}} \, dt, \;\; x \to +\infty$$

At first sight, it might seem that a direct application of Watson's lemma should produce an asymptotic expansion for the integral. Why is this not so? Use the change of variables $t\sqrt{x} = s$ to show that

$$\int_0^\infty e^{-xt} e^{-\frac{1}{t}} \, dt \sim x^{-\frac{3}{4}} \sqrt{\pi} e^{-2\sqrt{x}} \quad x \to +\infty$$

Exercise 3:

By using the change of variables $\tau = \log t$ and integrating by parts, show that as$x \to +\infty$

(a) $$\int_x^{\infty} \frac{dt}{t^2 \log t} \sim \frac{1}{x \log x}$$

(b) $$\int_2^{x} \frac{dt}{t \log(\log(t))} \sim \frac{\log x}{\log(\log(x))}$$

Exercise 4:

Use Watson's lemma to calculate the full Poincaré asymptotic expansion of

(a) $\displaystyle\int_0^\infty e^{-xt}\log(1+\sqrt{t})\,dt,\ \ x\to+\infty$

(b) $\displaystyle\int_0^\infty \frac{e^{-xt}}{\sqrt{t(2+t)}}\,dt,\ \ x\to+\infty$