King Fahd University of Petroleum & Minerals
Department of Mathematics and Statistics
Math592/Game Theory & Applications
Final Exam
Five (4 regular & 1 bonus) Questions, May 27^{th}, 2014 [1]

1 Short Questions (20 points)

State whether each of the following statements is true or false **(2 point)**. For each, explain your answer in (at most) a short paragraph, example or counter-example **(3 points)**.

(a) A proper Nash equilibrium is always perfect.

(b) Every Correlated equilibrium is a Nash equilibrium.

(c) Selten's chain store paradox was derived from the non-contradiction between the induction hypothesis and the deterrence hypothesis solutions.

(d) A player using a trigger strategy initially cooperates but punishes the opponent if a certain level of defection is observed.

[1] This is an open book exam. The exam game lasts 180 minutes.

2 Correlated equilibrium and Chicken Game (20 points)

Consider a version of the "Chicken game" described as follows. Like all forms of the chicken game, there are three Nash equilibria. The two pure strategy Nash equilibria are (D, C) and (C, D). There is also a mixed strategy equilibrium which results in expected payoffs of 14/3 = 4.667 for each player.

	D	C
D	$0, 0$	$7, 2$
C	$2, 7$	$6, 6$

Now consider a third party (or some natural event) that draws one of three cards labeled: (C, C), (D, C), and (C, D). This exogenous draw event is assumed to be uniformly at random over the 3 outcomes. After drawing the card the third party informs the players of the strategy assigned to them on the card (but not the strategy assigned to their opponent).

(a) Suppose a player is assigned D. Supposing the other player played their assigned strategy, would he want to deviate? (6 **points)**

(b) Suppose a player is assigned C. Supposing the other player played their assigned strategy, would he want to deviate? (6 **points)**

(c) Find the third Nash equilibrium in mixed strategies. Does is represent a coordinated equilibrium of this game? (8 **points)**

3 Smugglers vs Customs (30 points)

An import-export business company (X) is operating in the borders region between two countries. The company ships containers from country A to country B. The containers are loaded on large trucks and travel long distances to pass the borders. Due to the high levels of economic borders' activity between the two countries, the customs officers (Y) of the two countries perform common coordinated random inspections of the merchandise loaded in each container. The company (X) has the opportunity to realize huge profits if it succeeds in shipping large quantities of a precious metal to country B. This illegal but very profitable activity is called **smuggling**.

If the customs (Y) do inspect and catch the company (X) smuggling, they impose a large penalty payment $P > 0$ for this crime. The company (X) has to choose between two strategies; Ship (S) or Not-Ship (NS). The customs have to choose between two strategies; Inspect (I) or Not to inspect (NI). If the customs play (I) and the company plays (S), the customs get a payoff of P and the company gets a payoff of $-P$. If the customs play (I) and the company plays (NS), the customs get a payoff of -10 and the the company gets a payoff of 0. If the customs play (NI) and the company plays (S), the customs get a payoff of -100 and the company gets a payoff of 500. Finally, if the customs plays (NI) and the company play (NS), the customs get a payoff of 0 and the consumer gets a payoff of 0.

(a) For which values of P none of the pure strategies of (X) and (Y) is strictly dominated by the other. **(3 points)**

(b) Assume that $P = 1000$. Use the **Lemke & Howson's algorithm** to find a Nash equilibrium of this game. Is your Nash equilibrium quasi-strong? Isolated? Regular? **(**$4 + 2 + 1 + 1 = 8$ **points)**

(c) Assume that $P = 500$. Use the **Lemke & Howson's algorithm** to find a Nash equilibrium of this game. Is your Nash equilibrium perfect? Proper? Explain rigorously. **(**$3 + 1 + 1 + 3 = 8$ **points)**

(d) Discuss the impact of the change in the value of P on the strategies played by (X) and (Y). Does it confirm the deterrence hypothesis?**(**4 **points)**

(e) If the game was repeated infinitely, which **trigger strategy** should the customs choose, depending on the penalty P and the discount rate σ? **(7 points)**

4 Vehicle Routing Game (30 points)

A customer i bought some merchandise from a mega-store. The customer is waiting for it to be delivered to his home. On the given delivery day, he is part of a group of n customers scheduled to be visited by the p drivers of the transportation company. Suppose that the item(s) customer i purchased can be represented by a demand parameter D_i. Suppose also that the total capacity of a driver's truck can be represented by a capacity parameter C_j. The utility of customer i, for being served by driver j, is p_{ij}. The utility of driver j, for serving customer i, is q_{ij}. Suppose that every customer can be served by more than one driver. Suppose also that every truck can serve many customers. Finally, suppose that every customer and every driver can decide, respectively, which driver is serving him, and which customer(s) to serve. In the following, you are asked to model this problem as a strategic form game.

(a) Define the decision variable(s) for each customer i. **(3 points)**

(b) Define the decision variable(s) for each driver j. **(3 points)**

(c) Formulate the utility maximization program for each customer i. **(6 points)**

(d) Formulate the utility maximization program for each driver j. **(6 points)**

(e) Formulate the dual programs of the programs found in (c) and (d). **(6 points)**

(f) Find all the Nash equilibrium conditions. **(6 points)**

5 Perfect equilibrium? (Bonus 20 points)

Selten's definition of perfect equilibrium for a strategic form game can be stated as follows for a bimatrix game.

Definition 5.1 *Let $\hat{X} = (\hat{X}_1, \hat{X}_2)$ be a Nash equilibrium of a bimatrix game. The equilibrium $\hat{X}$ is perfect if there exists a sequence $\{X^r\}_{r\in\mathbb{N}} = \{(X_1^r, X_2^r)\}_{r\in\mathbb{N}}$ of completely mixed strategy vectors converging to $\hat{X} = (\hat{X}_1, \hat{X}_2)$, such that for all $r \in \mathbb{N}$ and $i = 1, 2$:*

$$\hat{X}_i \in CM_i\left[A_{i,-i}, X_{-i}^r\right].$$

In your home work 5, you were asked to show that in every perfect equilibrium, for any given player, any weakly dominated strategy should be assigned a zero probability. **Suppose that this result is true and that you can use it if needed**.

The following definition of a bimatrix game perfect equilibrium was presented during our lecture.

Definition 5.2 *Let $(\hat{x}_1, \hat{x}_2)$ be a Nash equilibrium of a bimatrix game $G(A, B)$. If there is a unit vector x_1 such that $x_1 A \geq \hat{x}_1 A$ and $x_1 A \neq \hat{x}_1 A$, or if there is a unit vector x_2 such that $Bx_2 \neq B\hat{x}_2$ and $Bx_2 \geq B\hat{x}_2$ then $(\hat{x}_1, \hat{x}_2)$ is not perfect. Otherwise, $(\hat{x}_1, \hat{x}_2)$ is said to be perfect.*

Show, rigorously, that definition 5.2 is equivalent to definition 5.1.