King Fahd University of Petroleum and Minerals
Department of Mathematics and Statistics

MATH 572, Semester 132

Assignment 1 Due Date: 19 March, 2014
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Consider the following two-point BVP:

$$-(a(x)\,u')' + c(x)u = f(x) \quad \text{for } x \in \Omega \text{ with } u(0) = u'(1) = 0, \tag{1}$$

where $\Omega = (0,1)$, and a, c and f are smooth functions with $a(x) > a_0 > 0$ and $c(x) \geq 0$ for $0 < x < 1$. Here u is a function of x.

a) Show that the BVP (1) has a unique weak solution in the Sobolev space

$$H(\Omega) := \{v \in H^1(\Omega) : \ v(0) = 0\}.$$

Choose $a(x) = c(x) = 1$ **and** $f(x) = (\pi^2/4 + 1)\sin(\pi x)$ in problem (1).

b) Verify that $u = \sin(\pi x/2)$ is the solution of (1).

c) Define the piecewise linear FEM of (1) over a uniform mesh consists of **M** subintervals of length **h** each.

d) Write the FE scheme in a matrix form (find the elements of the obtained matrices).

e) Choose $\mathbf{M} = 20$. Plot the exact solution and the FE solution on the same figure and compare between them.

f) Compute the L_2-error between the exact solution u and the piecewise linear FE solution, say $u_{\mathbf{h}}$, for $\mathbf{M} = 20, 40, 80, 160$. Find the order of convergence.

Choose $a(x) = 1$**,** $c(x) = x$, **and** $f(x) = (\pi^2/4 + x)\sin(\pi x)$ in problem (1).

g) Verify that $u = \sin(\pi x/2)$ is the solution of (1).

h) Define the piecewise linear FEM of (1) over a uniform mesh consists of **M** subintervals of length **h** each.

i) Compute the L_2-error between the exact solution u and the piecewise linear FE solution, say $u_{\mathbf{h}}$, for $\mathbf{M} = 20, 40, 80, 160$. Find the order of convergence.