King Fahd University of Petroleum & Minerals
Department of Math. & Stat.

Math 568 - Midterm Exam (132)
Time: 2 hours 00 mns

Saturday, April 4, 2014

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Name: ID #

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Problem 1	/10
Problem 2	/10
Problem 3	/5
Problem 4	/15
Total	/40

Problem # 1. (10 marks) Use the characteristic method to solve the problem

$$\begin{aligned} &u^2 u_x + u_y = 0, \quad x > 0 \\ &u(x,0) = \sqrt{x} \end{aligned} \qquad (i)$$

Problem # 2. (10 marks) Use the characteristic method to solve

$$u_x^2 + u_y^2 = 4, \qquad u_{|\Gamma} = 0$$

where

$$\Gamma = \{(x, y) = (2\cos\theta, 2\sin\theta),\ \theta \in \mathbb{R}\}$$

Problem # 3. (5 marks) Show that

$$\begin{aligned} -\Delta u(x) + 2\int_\Omega u(y)dy &= f(x), \qquad \text{in } \Omega \\ \frac{\partial u}{\partial \eta} &= \varphi, \qquad \text{on } \partial\Omega \end{aligned}$$

has at most one solution

Problem # 4. (15 marks) Given

$$u_{xx} + 2u_{xy}\cos x - u_{yy}\sin^2 x - u_y \sin x = 0, \qquad (*)$$

a. Show that the PDE (*) is hyperbolic
b. By an appropriate change of variables reduce $(*)$ to the canonical form and
b. Solve $(*)$ when $u(0, y) = y^2$ and $u_x(0, y) = -4y$.