⑤ Let $f:(E,\Sigma)\longrightarrow(\overline{\mathbb{R}},\mathcal{B}(\overline{\mathbb{R}}))$ be a function

The following are equivalent

(a) f is Σ-measurable

(b) $f^{-1}((a,+\infty])\in\Sigma \quad \forall\, a\in\mathbb{R}$

(c) $f^{-1}([a,+\infty])\in\Sigma \quad \forall\, a\in\mathbb{R}$

(d) $f^{-1}([-\infty,a))\in\Sigma \quad \forall\, a\in\mathbb{R}$

(e) $f^{-1}([-\infty,a])\in\Sigma \quad \forall\, a\in\mathbb{R}$.

⑥ Let $f,g\in M(E,\Sigma)$, then the following sets are measurable (that is they are in Σ)

i) $A=\{f>g\}$

ii) $B=\{f\geq g\}$

iii) $C=\{f=g\}$.

⑦ Let $f,g\in M(E,\Sigma)$, then $f+g,\ fg\in M(E,\Sigma)$.

⑧ Let $A\subset\mathbb{R}$, $A\neq\emptyset$ the following are equivalent

i) $A\in\mathcal{L}$

ii) $\forall\,\varepsilon>0\ \exists$ an open set $O\supset A$ such that $\lambda^*(O\setminus A)<\varepsilon$

iii) $\forall\,\varepsilon>0\ \exists$ a closed set $C\subset\mathbb{R}$ $C\subset A$ such $\lambda^*(A\setminus C)<\varepsilon$

iv) $\exists\ G\subset\mathbb{R}$, $G\in G_\delta$, $G\subset A$ such that $\lambda^*(G\setminus A)=0$

(v) $\exists\ H\subseteq\mathbb{R}$, $H\in F_\sigma$ $H\subset A$ such that $\lambda^*(A\setminus H)=0$.