① Let $(E, \Sigma, \mu)$ be a measure space, we define
$\overline{\Sigma} = \{ S \subset E : S = A \cup N,\ A \in \Sigma$ and $N \in \mathcal{N}_\mu \}$ and
$\overline{\mu} : \overline{\Sigma} \longrightarrow \overline{\mathbb{R}}_+$ by $\forall\ A \in \Sigma,\ \forall\ N \in \mathcal{N}_\mu\ \ \overline{\mu}(A \cup N) = \mu(A)$,
then

(a) $\overline{\Sigma}$ is a $\sigma$-algebra $\supset \Sigma$

(b) $\overline{\mu}$ is a complete positive measure extending $\mu$ from $\Sigma$ to $\overline{\Sigma}$

(c) The space $(E, \overline{\Sigma}, \overline{\mu})$ is the smallest complete measure space "containing" $(E, \Sigma, \mu)$

② Let $(E, \Sigma, \mu)$ be a measure space, and $(E, \overline{\Sigma}, \overline{\mu})$ its completion.
If $\mu^*$ is the outer measure generated by $\mu$ and $B_{\mu^*}$ the $\sigma$-algebra of $\mu^*$ measurable sets, then

$$\overline{\Sigma} \subset B_{\mu^*} \quad \text{and} \quad \mu^*\big|_{\overline{\Sigma}} = \overline{\mu}$$

③ Let $(E, \Sigma, \mu)$ be a finite measure space and $\mu^*$ is the outer measure extending $\mu$ to $\mathcal{P}(E)$, Prove that

$$A \in B_{\mu^*} \iff \mu^*(E) = \mu^*(A) + \mu^*(A^c)$$

④ Prove that

(i) If $A \subset \mathbb{R}$ is such that $\lambda^*(A) = 0$ then $A \in \mathcal{L}$

(ii) $m$ is complete

(iii) $\mathcal{B}(\mathbb{R}) \subset \mathcal{L} \subsetneq \mathcal{P}(\mathbb{R})$

(iv) $m(x + A) = m(A)\quad \forall\ x \in \mathbb{R},\ A \in \mathcal{L}$

(v) $m(xA) = |x|\, m(A)\quad \forall\ x \in \mathbb{R}\quad A \in \mathcal{L}$