(1) If $E = \mathbb{N}$, $\mathcal{A} = \{A \subset \mathbb{N} : |A| < \infty$ or $|A^c| < \infty\}$ Show that $\mathcal{A}$ is an algebra

(2) Prove the $\Sigma_\tau = \{A \subset \mathbb{R} : |A| < \omega_0$ and $|A^c| < \omega_0\}$ is a σ-algebra

(3) Define $\mathcal{A} = \{A \subset E : |A| < \infty$ or $|A^c| < \infty\}$ with E infinite, find $\sigma(\mathcal{A})$

(4) Let $\psi : \mathcal{C} \longrightarrow \mathbb{R} \cup \{+\infty\}$

$A, B \in \mathcal{C}$ $\ni$ $A \subset B$ and $B \setminus A \in \mathcal{C}$ and $\psi(A)$ is finite,

Prove that $\psi(B \setminus A) = \psi(B) - \psi(A)$.

(5) $E \neq \emptyset$ $\mu : \mathcal{P}(E) \longrightarrow [0, +\infty]$ define

$$\mu(A) = \begin{cases} |A| & \text{if } A \text{ is finite} \\ +\infty & \text{if } A \text{ is infinite} \end{cases}$$

Prove that this is a positive measure

(6) Prove that an outer-measure need not be a positive measure

(7) Let ~~[illegible]~~ $f : E \longrightarrow F$, $\mathcal{C} \subset \mathcal{P}(F)$, $\mathcal{C}_1, \mathcal{C}_2 \subset \mathcal{P}(E)$ such that $\mathcal{C}_1 \subset \mathcal{C}_2$ then

(a) $\mathcal{C}_1 \subset a(\mathcal{C}_1) \subset a(\mathcal{C}_2)$

$\mathcal{C}_1 \subset \sigma(\mathcal{C}_1) \subset \sigma(\mathcal{C}_2)$

(b) $\sigma_E(f^{-1}(\mathcal{C}_1)) = f^{-1}(\sigma_F(\mathcal{C}))$

(8) Let $E \neq \emptyset$ and $\mathcal{C} \subset \mathcal{P}(E)$, $\mathcal{C} \neq \emptyset$ then $\exists$ a smallest algebra (σ-algebra) containing $\mathcal{C}$.