(VI) Let $f : \mathbb{R} \to \mathbb{R}$ be defined by

$$f(x) = \begin{cases} x^2 \sin \frac{1}{x^2}, & x > 0 \\ 0, & x \leq 0. \end{cases}$$

Prove that $f \chi_{(0,a)} \notin \mathcal{L}^1(\mathbb{R}, \mathcal{B}(\mathbb{R}), \lambda)$, $a > 0$.

(VII) State and prove the Minkowski inequality.