KFUPM, DEPARTMENT OF MATHEMATICS AND STATISTICS

MATH 521: MIDTERM EXAM, TERM (132), APRIL 26, 2014

Time: 09:00(am) to 11:00

Name : ....

ID : .... Section : ....

Exercise 1. Let A, B be subsets of a metric space (X, d) such that $\overline{A} \cap B = A \cap \overline{B} = \emptyset$. Show that there exist two disjoint open sets U, V containing respectively A and B.

Exercise 2. Let X, Y be topological spaces and $A \subseteq X, B \subseteq Y$. Show that

$$\mathrm{Fr}(A \times B) = (\mathrm{Fr}(A) \times \overline{B}) \cup (\overline{A} \times \mathrm{Fr}(B)).$$

Exercise 3. Let X, Y be topological spaces and $f : X \longrightarrow Y$ be a function and $G = \{(x, y) \in X \times Y : y = f(x)\}$ be the graph of f. Show that f is continuous if and only if the function $g : G \longrightarrow X$ induced by first projection is a homeomorphism.

Exercise 4. Find a closed set C of $\mathbb{R} \times \mathbb{R}$ such that the first projection $p_1(C)$ is not closed in $\mathbb{R}$.

Exercise 5.

(a) Give an example of subsets of $\mathbb{R}$ such that

$$\operatorname{int}(A) \cup \operatorname{int}(B) \subset \operatorname{int}(A \cup B).$$

(b) Let A, B be subsets of a space X such that $A \cap \overline{B} = \overline{A} \cap B = \emptyset$. Show that

$$\operatorname{int}(A) \cup \operatorname{int}(B) = \operatorname{int}(A \cup B).$$

Exercise 6. Let X be a topological space and Y be a subset of X.

(a) Show that $\overline{X \setminus Y} = X \setminus \operatorname{int}(Y)$.

(b) Let A be a subset of X. Show that A is open if and only if

$$\overline{A \cap \overline{B}} = \overline{A \cap B},$$

for each subset B of X.

Exercise 7. Consider the quotient space $\mathbb{R}/\mathbb{Q}$, where the equivalence relation on $\mathbb{R}$ is defined by

$$x \equiv y \text{ if and only if } x - y \in \mathbb{Q}.$$

Show that the space $\mathbb{R}/\mathbb{Q}$ is indiscrete.

Exercise 8. Let $f : X \longrightarrow Y$ be a surjective continuous map. Show that if f is an open(resp., closed) map, then it is a quotient map.

Exercise 9. Let $\equiv$ be the equivalence relation defined on $\mathbb{R}$ by:

$$x \equiv y \text{ if and only if } x - y \in \mathbb{Z}.$$

We denote by T^1 the quotient space $\mathbb{R}/\equiv$.
Give an explicit continuous bijection from T^1 into the unit circle S^1.