King Fahd University of Petroleum & Minerals
Department of Mathematics & Statistics
Math 513 Final Exam
The Second Semester of 2013-2014 (132)
Time Allowed: 180 Minutes

Name: ______________________ ID#: ______________

Instructor: ______________________ Sec #: ________ Serial #: ________

- Mobiles and calculators are not allowed in this exam.
- Write all steps clear.

Question #	Marks	Maximum Marks
1		16
2		16
3		14
4		20
5		20
6		14
Total		100

Q:1 (16 points) Solve the heat equation

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}, \quad 0 < x < \pi, \quad t > 0$$

subject to the following initial and **nonhomogeneous** boundary conditions

$u(x, 0) = 4$ for $0 < x < \pi$ and $u(0, t) = 0, \quad u(\pi, t) = 4$ for $t > 0$.

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Q:2 (16 points) Use Laplace transformation method to solve the wave equation

$$\frac{\partial^2 u}{\partial t^2} - \frac{\partial^2 u}{\partial x^2} = te^{-x}, \quad 0 < x < \infty, \quad t > 0,$$

with the initial conditions $u(x,0) = 0$, $u_t(x,0) = x$, for $0 < x < \infty$

and the boundary conditions $u(0,t) = 1 - e^{-t}$, $\lim_{x\to\infty} |u(x,t)| \sim x^n$, for some finite n, $t > 0$.

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Q:3 (14 points) Find steady–state temperature in a semi infinite plate by solving

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0, \quad 0 < x < \pi, \quad y > 0$$

subject to the following boundary conditions $u(0, y) = 0$, $u(\pi, y) = 0$ for $y > 0$

and $u(x, 0) = x$, $0 < x < \pi$. Also solution is bounded at $y \to \infty$.

Q:4 (20 points) Find the steady–state temperature in a hemisphere of radius 2 by solving

$$\frac{\partial^2 u}{\partial r^2} + \frac{2}{r}\frac{\partial u}{\partial r} + \frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2} + \frac{\cot\theta}{r^2}\frac{\partial u}{\partial \theta} = 0, \quad 0 < r < 2, \quad 0 < \theta < \frac{\pi}{2}$$

when the base of the hemisphere is insulated $\left[u_\theta(r, \frac{\pi}{2}) = 0\right]$

and $u(2, \theta) = \sin(\theta)$, $0 < \theta < \dfrac{\pi}{2}$. Find first three nonzero terms of the series solution.

(Hint: $P_n'(0) = 0$ only for even values of n)

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Q:5 (20 points) Find the displacement $u(x, t)$ in a circular plate of radius 2 by solving

$$\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} = \frac{\partial^2 u}{\partial t^2}, \quad 0 < r < 2, \quad t > 0$$

with initial conditions $u(r, 0) = r^2$, $u_t(r, 0) = 0$, $0 < r < 2$,

and the boundary condition $u(2, t) = 0$, $t > 0$. Solution is bounded at $r = 0$.

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Q:6 (14 points) Solve the nonhomogeneous linear system using variation of parameters method

$$X' = \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix} X + \begin{bmatrix} e^t \cos t \\ e^t \sin t \end{bmatrix}, \text{ with } X(0) = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$