King Fahd University of Petroleum & Minerals
Department of Mathematics & Statistics
Math 513   Major Exam 1
The Second Semester of 2013-2014 (132)

<u>**Time Allowed: 120 Minutes**</u>

Name: _____ ID#: _____

Instructor: _____ Sec #: _____ Serial #: _____

- Mobiles and calculators are not allowed in this exam.

- Write all steps clear.

Question #	Marks	Maximum Marks
1		12
2		12
3		12
4		12
5		12
6		14
7		10
8		16
Total		100

Q:1 (12 points) Find the Fourier series of $f(x) = x \cos\left(\dfrac{\pi x}{L}\right)$ for $-L < x < L$. Write the series in phase angle form.

Q:2 (12 points) Find the complex Fourier series of $f(x) = |x|$ for $-\pi \leq x \leq \pi$.

Q:3 (12 points) Show that Fourier transform of $f(x) = \begin{cases} \cos(ax) & |x| < 1 \\ 0 & |x| > 1 \end{cases}$,

is $F(w) = \dfrac{\sin(w - a)}{w - a} + \dfrac{\sin(w + a)}{w + a}$.

Q:4 (12 points) Let $f(t) = H(t) - H(t - 2)$ and $g(t) = e^t H(-t)$.

Show that $f(t) * g(t) = \begin{cases} e^t - e^{t-2} & t \leq 0 \\ 1 - e^{t-2} & 0 \leq t \leq 2 \\ 0 & 2 \leq t \end{cases}$.

Q:5 (12 points) Solve the initial value problem by using Laplace transform,

$$y'' + 3y' + 2y = t + [ae^{-(t-a)} - t]H(t - a), \ y(0) = y'(0) = 0.$$

Q:6 (7+7 points) (a) Use convolution to find inverse laplace transform of $F(s) = \dfrac{1}{(s^2 + 4)^2}$.

(b) Verify the convolution theorem for $t * [H(t) - H(t-2)] = \dfrac{t^2}{2} - \dfrac{(t-2)^2}{2}H(t-2)$

by showing that Laplace transforms of two sides are same.

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Q:7 (10 points) Solve the integral equation $f(t) = 3t^2 - e^{-t} - \int\limits_0^t f(\tau)e^{t-\tau}d\tau$ for $f(t)$.

Q:8 (8+8 points) (a) Find eigenvalues and eigenfunctions of

$$y'' + \lambda y = 0, \ y'(0) = 0 \text{ and } y'(\pi) = 0.$$

(b) Show that these eigenfunctions are orthogonal. Also find eigenfunction expansion of

$f(x) = x$ using these eigenfunctions.

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