King Fahd University of Petroleum and Minerals
Department of Mathematics and Statistics
MATH430 - Introduction to Complex Variables
Exam I – Term 132 (2013–2014)

Exercise 1 (5 points)

Solve the equation $(z+1)^5 = z^5$.

Exercise 2 (`10 points`)

Find *all* the values of

(a) $\left(\dfrac{2i}{1+i}\right)^{1/6}$,

(b) $|(-i)^{-i}|$.

Exercise 3 (8 points)

Find the image of the horizontal line $y = c$, $c > 0$ under the mapping $w = \frac{1}{z}$.

Exercise 4 (12 points)

(a) Show that $cos^{-1}(z) = -i\log\left(z + i(1-z^2)^{1/2}\right)$.

(b) Solve $cos(z) = 2$.

(c) Find the principal value of $cos^{-1}(x)$, $x \in [-1,1]$.

(d) Find the branch of $cos^{-1}(x)$ using the principal log and log_0 for the root, where $x \in [-1,1]$.

Exercise 5 (`7 points`)
Find an analytic function $f(z)$ such that

$$\Re f'(z) = 3x^2 - 4y - 3y^2 \text{ and } f(1+i) = 0.$$

Exercise 6 (8 points)

1. If u and v are harmonic in a region Ω, prove that

$$F = \left(\frac{\partial u}{\partial y} - \frac{\partial v}{\partial x}\right) + i\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right)$$

   is analytic in Ω.

2. Assume that $u = \Re f$ and $v = \Re g$, where f and g are analytic functions in Ω. Find F in terms of f and g.