King Fahd University of Petroleum and Minerals
Department of Mathematics & Statistics
Math 425 Final Exam Spring 2014(132)

ID#:________________ NAME:________________________________

Total Score(out of 140)#________________ Time allowed: 150 minuts

NO CREDITS WILL BE GIVEN FOR ANSWERS NOT SUPPORTED BY WORK

PART I: (90 POINTS)

1. Define each of the following terms: (20pts)
   (a) Irregular graph
   (b) Self-complementary graph
   (c) Tournament
   (d) Eccentricity of a vertex
   (e) An (r, n)-cage
   (f) A block graph
   (g) A maximal planar graph
   (h) An automorphism of a graph G
   (i) The graph orbits
   (j) Moore graph
2. State each of the following theorems: (12pts)
   (a) The first theorem of graph theory.
   (b) The matrix-tree theorem
   (c) Robbins' theorem
   (d) Frucht's theorem
   (e) Kuratowski's theorem
   (f) Tur*á*n's theorem
3. Consider the graph $G = K_{3,4}$. Answer each of the following. If your answer is no, explain why; and if yes, support it by construction or calculation. (14pts)
   (a) Is G Eulerian graph?
   (b) Is G Hamiltonian graph?
   (c) Is G separable?

(d) Find the number of bridges of G.

(e) Find the crossing number $cr(G)$.

(f) Find the edge-connectivity $\lambda(G)$.

(g) Find the independence number $\alpha(G)$.

4. Give four different equivalent conditions for a tree. (3pts)

5. Is there a graph with degrees $1, 1, 3, 3, 3, 3, 5, 6, 8, 9$? (3pts)

6. Is there a bipartite graph with degrees $3, 3, 3, 3, 3, 5, 6, 6, 6, 6, 6, 6, 6, 6$? (3pts)

7. Give a necessary condition for a Hamiltonian graph. (4pts)

8. Let G be a graph on 8 vertices. If G is 3-connected, how many edges must G have? (3pts)

9. Let T be a transitive tournament on n ($n \geq 3$) vertices. (10pts)

   (a) How many directed Hamiltonian paths does T contain?

   (b) How many directed Hamiltonian circuits does T contain?

   (c) Find the score sequence of T.

   (d) How many labeled spanning trees does the underlying graph T_u have?

   (e) Show that T is acyclic.

10. For $n \geq 4$, determine $ex(n; K_{1,3})$ and all extremal graphs. (5pts)

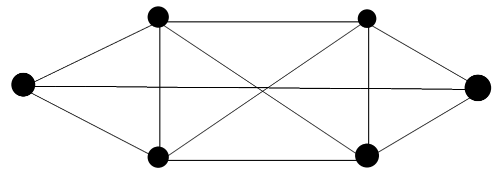
11. Recall Dirac's theorem: "If $1 \leq K \leq n \Rightarrow d_k \geq \frac{n}{2}$, then G is Hamiltonian". Show by an example that Dirac's theorem is no longer true if $\frac{n}{2}$ is replaced by $\frac{n-1}{2}$. (4pts)

12. (a) Let G be a connected planar graph with $n \geq 3$ vertices, m edges and girth $g = 5$. Show that $m \leq \frac{5}{3}(n-2)$. (4pts)

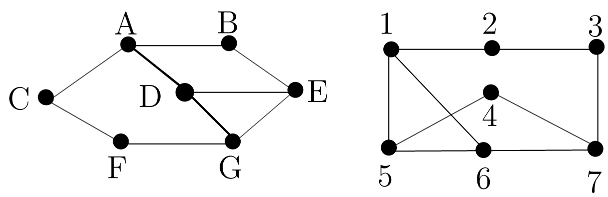
    (b) Use (a) to show that the Peterson graph is non-planar. (5pts)

PART II: (50 POINTS-5pts Each)
Either prove or disprove each of the following statements:

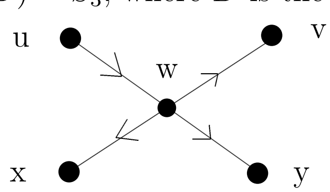
1. The graph in the figure is planar.



2. The following two graphs are isomorphic.



3. If G_1 and G_2 are bipartite graphs, then $G_1 \vee G_2$ is bipartite.
4. If G is a maximal planar graph, then $\delta(G) \leq 5$.
5. If $H = G \vee \{u\}$, where G is n-connected, then H is $(n+1)$-connected.
6. $Aut(D) \cong S_3$, where D is the digraph below



7. There are at least two 5-cages.
8. $n(3,6) = 14$, the smallest order of an $(3,6)$-graph is 14.
9. $K(3,3)$ is nonplanar.
10. Every n-cycle is a Cayley graph.

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