King Fahd University of Petroleum and Minerals
Department of Mathematics & Statistics
Math 425 Exam III Spring 2014(132)

ID#:________________ NAME:________________________________

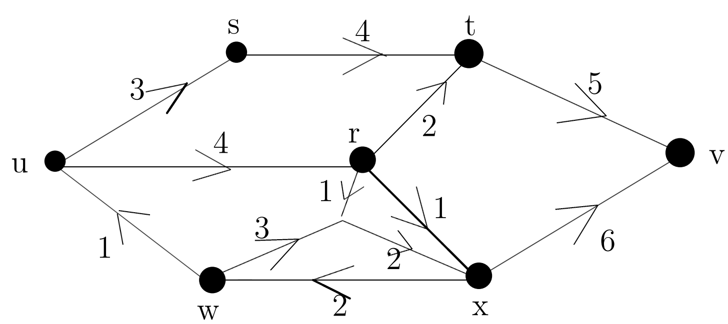
Total Score#________________
NO CREDITS WILL BE GIVEN FOR ANSWER WITHOUT EXPLANATION.

---

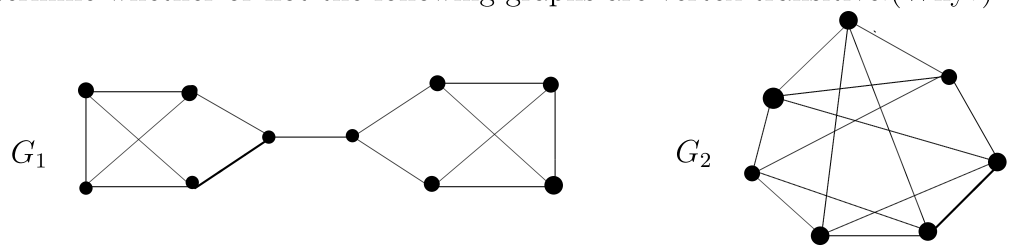
(1) (a) Prove that if f is flow in a network N with capacity function c and $[X, \overline{X}]$ is a cut in N such that $f(a) = c(a)$ for all $a \in [X, \overline{X}]$ and $f(a) = 0$ for all $a \in [\overline{X}, X]$, then f is a maximum flow in N and $[X, \overline{X}]$ is a minimum cut.

(b) Use the Ford-Fulkerson algorithm to find a maximum flow f and a minimum cut K in the network N below:

N:

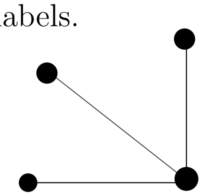


(2) Determine whether or not the following graphs are vertex transitive.(Why?)



(4) For the graph G find:
(a) its automorphism group $Aut(G)$ (b) its orbits (c) the number of distinct labelings of G from a set of 4 labels.

G:



(3) Construct the cayley color graph of the dihedral group $\mathbb{D}_4 = \{e, a = (1\ 2\ 3\ 4), (1\ 3)(2\ 4), (1\ 4\ 3\ 2), b = (1\ 3), (2\ 4), (1\ 2)(3\ 4), (1\ 4)(2\ 3)\}$, when the generating set $\Delta = \{a, b\}$.

(5) (a) Prove that every planar graph must contain a vertex of degree at most 5.
(b) Show that if G is a planar graph containing no vertex of degree less than 5, then G contains at least 12 vertices of degree 5.
(c) Give an example of two non-isomorphic maximal planar graphs of the same order.
(d) What values of n and m imply that $K_{n,m}$ is planar? For each case compute the number of regions.

Dr. M. R. Alfuraidan