

King Fahd University of Petroleum and Minerals  
Department of Mathematical Sciences  
**Math 425 Exam II Spring 2013(032)**

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NAME: \_\_\_\_\_

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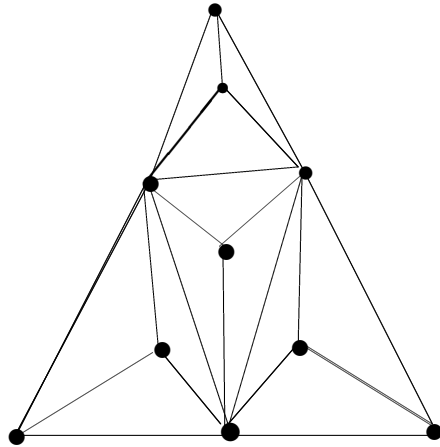
(1) (a) Define:

(i) an Eulerian circuit (ii) edge connectivity (iii)  $k$ -connected graph (iv) a strong digraph

(b) State:

(i) Menger's Theorem (ii) Whitney's Theorem (iii) Robbin's Theorem

- (2) Consider the graph  $G$  in the figure below:
- (a) Either construct a Hamiltonian cycle or prove that  $G$  is not Hamiltonian.
  - (b) What is the minimum number of edges we need to add to make  $G$  an Eulerian?
  - (c) Is it possible to direct the edges of  $G$  to make it strongly connected? How?



(3) For what value(s) of  $n$  are the following graphs Hamiltonians and for what value(s) of  $n$  are they Eulerians?

(a)  $K_{n,2n,3n}$                       (b)  $K_{n,2n,3n+1}$

- (5) Prove each of the following statements:
- (a) A tournament is strongly connected if and only if it contains a Hamiltonian cycle.
  - (b) If  $G$  is Eulerian graph, then every cut-set  $S$  of  $G$  must have an even number of edges.
  - (c) If  $G$  is a graph of order  $n \geq 2$  such that for all distinct nonadjacent vertices  $u, v$   $deg_u + deg_v \geq n - 1$ , then  $\lambda(G) = \delta(G)$ .

(6) Find (the connectivity)  $\kappa(G)$  and (independence number)  $\alpha(G)$  if  $G$  is:  
(a)  $K_n$             (b)  $K_{s,t}$             (c)  $C_n$             (d)  $P_n$

(7) Let  $G$  be a graph on 8 vertices. If  $G$  is 2-connected, how many edges must  $G$  have? Show why?