King Fahd University of Petroleum and Minerals
Department of Mathematical Sciences
Math 425 Exam1 Spring 2013(132)

ID#:________________ NAME:______________________________

Total Score#________________

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(1) Consider the disconnected graph $G = G_1 \cup G_2$. (3pts each)

(a) Find a subgraph that is not an induced subgraph of G.

(b) Find the radius and the diameter of G_1.

(c) Is G bipartite? If yes, give a bipartition. If not, explain why not.

(d) How many edges does the complement $\overline{G}$ have?

(e) Find all cut-vertices and bridges of G.

(f) Find a spanning connected (component-wise) subgraph having a minimum number of edges.

(g) Find all blocks of G.

(2) If A is the adjacency matrix of a graph G and (16pts)

$$A^2 = \begin{pmatrix} 4 & 1 & 1 & 4 & 1 & 1 \\ 1 & 3 & 2 & 1 & 2 & 2 \\ 1 & 2 & 3 & 1 & 2 & 2 \\ 4 & 1 & 1 & 4 & 1 & 1 \\ 1 & 2 & 2 & 1 & 3 & 2 \\ 1 & 2 & 2 & 1 & 2 & 3 \end{pmatrix}, \ A^3 = \begin{pmatrix} 4 & 9 & 9 & 4 & 9 & 9 \\ 9 & 4 & 5 & 9 & 4 & 4 \\ 9 & 5 & 4 & 9 & 4 & 4 \\ 4 & 9 & 9 & 4 & 9 & 9 \\ 9 & 4 & 4 & 9 & 4 & 5 \\ 9 & 4 & 4 & 9 & 5 & 4 \end{pmatrix},$$

(a) How many walks are there between v_1 and v_6 of length 3?

(b) Find the degree sequence of G?

(c) Find the number of triangles of G.

(d) Draw the graph G.

(e) Suppose that $d_1, d_2, ..., d_6$ is the degree sequence, in nondecreasing order, of G, labeled its vertices as v_1,v_2,v_3,v_4,v_5,v_6, then find the number of its distinct spanning trees.

(3) Determine if the following statements are TRUE or FALSE. If a statement is true, sketch the proof; if it is false, give a counterexample. (16pts)

(a) The following two graphs are isomorphic.

(b) If G is connected, then $\overline{G}$ is disconnected.

(c) If the number of edges in a graph is less than the number of vertices, then the graph must be disconnected.

(d) No digraph contains an odd number of vertices of odd outdegree or an odd number of vertices of odd indegree.

(4) A tree is called central if its center is K_1 and bicentral if its center is K_2. Show that every tree is central or bicentral. (5pts)

(5) Show that if G is an r-regular connected graph, where r is even, then G contains no bridges. (5pts)

(6) Let G be a nontrivial connected graph such that each vertex is of even degree. Show that G has at least one circuit. (5pts)

(7) (a) Determine all nonisomorphic graphs of order 4. Which one(s) are self-complementary?
. (7pts)

(b) Suppose that $1, 1, 1, 3, 3, 4, 5, 6, 6, 7, x, 8$ is the degree sequence, in nondecreasing order, of a graph. Determine x. (5pts)

(c) Prove that there exist regular tournament of every odd order but there are no regular tournament of even order. (5pts)

(8) Prove that every nontrivial connected graph contains at least two vertices that are not cut-vertices. (5pts)

(b) Determine the labeled tree having Prüfer code $\{4, 5, 7, 2, 1, 1, 6, 6, 7\}$. (5pts)

(7) Using Kruskal's algorithms to find a minimum spanning tree in the following weighted graph. (5pts)

Dr. M. R. Alfuraidan