King Fahd University of Petroleum and Minerals
Department of Mathematics and Statistics
Math-301 Semester-132 QUIZ IV

NAME: S.No. ID:

Maximum Marks: 10 Section:06 Time Allowed: 30 minutes

(1) Show that the set $\{sinx, sin3x, sin5x.........\}$ is orthogonal on the interval $[0, \frac{\pi}{2}]$.

(2) Expand

$$f(x) = \begin{cases} 0, & -1 < x < 0, \\ x, & 0 \le x < 1. \end{cases}$$

in a Fourier series.

(3) Expand $f(x) = x^2, \quad 0 < x < L$ in a cosine series.

Sol.1 : $\phi_n(x) = \sin(2n+1)x, \quad \phi_m(x) = \sin(2m+1)x$

Now $\int_0^{\frac{\pi}{2}} \sin(2n+1)x \sin(2m+1)dx$ (1) $= \frac{1}{2}\int_0^{\pi/2}[\cos 2(n-m)x - \cos 2(n+m+1)]dx$

$= \left[\frac{\sin 2(n-m)x}{4(n-m)} - \frac{1}{4(n+m+1)}\sin 2(n+m+1)x\right]_0^{\pi/2}$ (1) $= 0 \Rightarrow$ Set of functions is orthogonal

Sol.2 : $a_0 = \int_{-1}^1 f(x)dx = \int_{-1}^0 0\,dx + \int_0^1 x\,dx = 0 + \frac{1}{2} = \frac{1}{2}$ (1)

$a_n = \int_{-1}^1 f(x)\cos n\pi x\,dx = \int_0^1 x\cos n\pi x\,dx = \left(\frac{x\sin n\pi x}{n\pi}\right)_0^1 - \frac{1}{n\pi}\int_0^1 \sin n\pi x\,dx$

$= \frac{1}{n^2\pi^2}\left[\cos n\pi x\right]_0^1 = \frac{1}{n^2\pi^2}\left[(-1)^n - 1\right]$ (2)

$b_n = \int_{-1}^1 f(x)\sin n\pi x\,dx = \int_0^1 x\sin n\pi x\,dx = \frac{1}{n\pi}\left[-x\cos n\pi x\right]_0^1 + \int_0^1 \frac{\cos n\pi x}{n\pi}dx$

$= \frac{1}{n\pi}\left[(-1)(-1)^n\right] + \frac{1}{n^2\pi^2}\left[\sin n\pi x\right]_0^1 = \frac{(-1)^{n+1}}{n\pi} + 0$ (2)

Thus $f(x) = \frac{1}{4} + \sum_{n=1}^{\infty}\left[\frac{(-1)^n - 1}{n^2\pi^2}\cos n\pi x + \frac{(-1)^{n+1}}{n\pi}\sin n\pi x\right]$ (1)

Sol.3 : $a_0 = \frac{2}{L}\int_0^L x^2 dx = \frac{2}{3}L^2$ $\left(\frac{1}{2}\right)$

$a_n = \frac{2}{L}\int_0^L x^2\cos\frac{n\pi x}{L}dx = \frac{2}{L}\left[x^2\sin\frac{n\pi x}{L}\cdot\frac{L}{n\pi}\right]_0^L - \frac{2}{L}\cdot\frac{L}{n\pi}\int_0^L 2x\sin\frac{n\pi x}{L}dx$

$= -\frac{4}{n\pi}\int_0^L x\sin\frac{n\pi x}{L}dx = \frac{4}{n\pi}\left[\frac{L}{n\pi}x\cos\frac{n\pi x}{L}\right]_0^L - \frac{4}{n\pi}\int_0^L \frac{L}{n\pi}\cos\frac{n\pi x}{L}dx$

$= \frac{4L^2(-1)^n}{n^2\pi^2} - 0$ (2)

Hence

$f(x) = \frac{L^2}{3} + \frac{4L^2}{\pi^2}\sum_{n=1}^{\infty}\frac{(-1)^n}{n^2}\cos\frac{n\pi x}{L}$. $\left(\frac{1}{2}\right)$