King Fahd University of Petroleum and Minerals
Department of Mathematics and Statistics
Math-301 Semester-132 QUIZ IV

NAME: S.No. ID:

Maximum Marks: 10 Section:03 Time Allowed: 30 minutes

(1) Show that the set $\{1, cosx, cos2x, ...\}$ is orthogonal on the interval $[-\pi, \pi]$.

(2) Expand

$$f(x) = \begin{cases} 0, & -\pi < x < 0, \\ \pi - x, & 0 \le x < \pi. \end{cases}$$

in a Fourier series.

(3) Expand $f(x) = x^2, \quad 0 < x < L$ in a cosine series.

Sol.1: $\phi_0(x) = 1, \quad \phi_n(x) = \cos nx$

$$(\phi_0, \phi_n) = \int_{-\pi}^{\pi} \cos nx \, dx = \left[\frac{1}{n}\sin nx\right]_{-\pi}^{\pi} = \frac{1}{n}\left[\sin n\pi - \sin(-n\pi)\right] = 0 \quad (1)$$

Sol.2: $$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\,dx = \frac{1}{\pi}\left[\int_{-\pi}^{0} 0\,dx + \int_0^{\pi}(\pi - x)\,dx\right]$$

$$= \frac{1}{\pi}\left(\pi x - \frac{x^2}{2}\right)_0^{\pi} = \frac{\pi}{2} \quad (1)$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nx\,dx = \frac{1}{\pi}\int_0^{\pi}(\pi - x)\cos nx\,dx$$

$$= \frac{1}{\pi}\left[(\pi - x)\frac{\sin nx}{n}\right]_0^{\pi} + \frac{1}{\pi n}\int_0^{\pi}\sin nx\,dx = 0 - \frac{1}{n^2\pi}\left[\cos nx\right]_0^{\pi} = \frac{1-(-1)^n}{n^2\pi} \quad (2)$$

$$b_n = \frac{1}{\pi}\int_0^{\pi}(\pi - x)\sin nx\,dx = \frac{1}{\pi}\left[(\pi - x)\frac{\cos nx}{n}\right]_0^{\pi} - \frac{1}{n\pi}\int_0^{\pi}\cos nx\,dx$$

$$= \frac{1}{n} - \frac{1}{n\pi}\left[\frac{\sin nx}{n}\right]_0^{\pi} = \frac{1}{n} \quad (1)$$

Hence
$$f(x) = \frac{\pi}{4} + \sum_{n=1}^{\infty}\left\{\frac{1-(-1)^n}{n^2\pi}\cos nx + \frac{1}{n}\sin nx\right\} \quad (1)$$

Sol.3: $$a_0 = \frac{2}{L}\int_0^L x^2\,dx = \frac{2}{3}L^2 \quad \left(\frac{1}{2}\right)$$

$$a_n = \frac{2}{L}\int_0^L x^2\cos\frac{n\pi x}{L}\,dx = \frac{2}{L}\left[x^2\sin\frac{n\pi x}{L}\cdot\frac{L}{n\pi}\right]_0^L - \frac{2}{L}\int_0^L \frac{2xL}{n\pi}\sin\frac{n\pi x}{L}\,dx$$

$$= -\frac{4}{n\pi}\int_0^L x\sin\frac{n\pi x}{L}\,dx = +\frac{4}{n\pi}\left[\frac{xL}{n\pi}\cos\frac{n\pi x}{L}\right]_0^L - \frac{4}{n\pi}\int_0^L\frac{L}{n\pi}\cos\frac{n\pi x}{L}\,dx$$

$$= \frac{4L^2(-1)^n}{n^2\pi^2} - 0 \quad (2)$$

Thus
$$f(x) = \frac{L^2}{3} + \frac{4L^2}{\pi^2}\sum_{n=1}^{\infty}\frac{(-1)^n}{n^2}\cos\frac{n\pi}{L}x. \quad \left(\frac{1}{2}\right)$$