King Fahd University of Petroleum and Minerals
Department of Mathematics and Statistics
Math-301 Semester-132 QUIZ I

NAME: S.No. ID:

Maximum Marks: 10 Section:03 Time Allowed: 40 minutes

(1) If $u = x^2y$ and $v = xz^2 - 2y$, then find $grad[(grad\ u).(grad\ v)]$.

(2) If $\mathbf{F}(x, y, z) = xye^z\mathbf{i} + yze^x\mathbf{j} + xze^y\mathbf{k}$, find $curl\ \mathbf{F}$ and $div(curl\ \mathbf{F})$.

(3) Evaluate $\int_C xy\ dx - x^2\ dy$, where C is given by $y = x^3, -1 \leq x \leq 2$.

(4) Evaluate $\int_C xyz\ dx - tan(yz)\ dy + xz\ dz$ over the straight line segment from $(1, 1, 1)$ to $(-2, 1, 3)$.

Sol.1: $\text{grad}\,u = \langle 2xy, x^2, 0\rangle$; $\text{grad}\,v = \langle z^2, -2, 2xz\rangle$

$$\text{grad}\,u \cdot \text{grad}\,v = 2xyz^2 - 2x^2$$

$$\text{grad}[\text{grad}\,u \cdot \text{grad}\,v] = \langle 2yz^2 - 4x, 2xz^2, 4xyz\rangle$$

Sol.2: $$\nabla \times \vec{F} = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ xye^z & yze^x & xze^y \end{vmatrix} = \langle xze^y - ye^x, xye^z - ze^y, yze^x - xe^z \rangle$$

$$\nabla \cdot (\nabla \times \vec{F}) = ze^y - ye^x + xe^z - ze^y + ye^x - xe^z = 0.$$

Sol.3: $$\int_C x(x^3)\,dx - x^2 \cdot 3x^2\,dx = \int_{-1}^{2} -2x^4\,dx = \left[-2\frac{x^5}{5}\right]_{-1}^{2}$$

$$= -\frac{2}{5}[32+1] = -\frac{66}{5}$$

Sol4: Parametric equations of the line through $(1,1,1)$ & $(-2,1,3)$ are

$x = 1-3t$, $y = 1$, $z = 1+2t$.

For initial and terminal points, t vary from 0 to 1.

$$\int_C xyz\,dx - \tan(yz)\,dy + xz\,dz$$

$$= \int_0^1 \left[(1-3t)(1)(1+2t)(-3) - \tan(1+2t)\cdot 0 + (1-3t)(1+2t)\cdot 2\right] dt$$

$$= \int_0^1 (6t^2 + t - 1)\,dt = \left[6\frac{t^3}{3} + \frac{t^2}{2} - t\right]_0^1$$

$$= 2 + \frac{1}{2} - 1 = \frac{3}{2}$$