KFUPM, DEPARTMENT OF MATHEMATICS AND STATISTICS

MATH 280: TESTS 3-4, SEMESTER (132), MAY 04, 2014

Time: 18:30 to 20:00

Name : ....

ID : .... Section : ....

Exercise	**Points**
1	____ / 12
2	____ / 12
3	____ / 12
4	____ / 12
5	____ / 14
6	____ / 12
7	____ / 14
8	____ / 12
Total	____ / 100

Exercise 1 (12 pts)**.** Is the set of all triples of real numbers (x, y, z) equipped with the standard vector addition but with scalar multiplication defined by $k(x, y, z) = (k^2x, k^2y, k^2z)$ a vector space (on $\mathbb{R}$)?

Exercise 2 (12 pts)**.** Determine which of the following subsets of $\mathbb{R}^3$ are subspaces of $\mathbb{R}^3$.

(1) All vectors of the form $(a, 0, 0)$.

(2) All vectors of the form $(a, 1, 1)$.

(3) All vectors of the form (a, b, c), where $b = a + c$.

(4) All vectors of the form (a, b, c), where $b = a + c + 1$.

Exercise 3 (12 pts)**.** Let

$$V = \left\{(x, y, z) \in \mathbb{R}^3 : 3x - y + 5z = 0\right\}.$$

Find a basis of V and evaluate the dimension of V.

Exercise 4 (12 pts)**.** Use the Wronskian to show that the functions

$$f_1(x) = e^x, f_2(x) = xe^x \text{ and } f_3(x) = x^2e^x$$

are linearly independent vectors in the space $\mathbf{C}^3(\mathbb{R})$.

Exercise 5 (14 pts).

(1) Find the transition matrix from $\mathcal{B} = (1, x, x^2)$ to the basis basis $\mathcal{B}' = (1, 1 + x, (1 + x)^2)$.

(2) Given any $p(x) = a + bx + cx^2$ in $\mathbf{P}_2$, find the coordinates of $p(x)$ with respect to the basis $\mathcal{B}' = (1, 1 + x, (1 + x)^2)$.

Exercise 6 (12 pts)**.** Let

$$A = \begin{pmatrix} 1 & 2 & -1 & 1 \\ 1 & 4 & -3 & 0 \\ 1 & 1 & 1 & 5 \end{pmatrix}$$

Find a basis for the row space $\mathbf{RS}(A)$ of A, a basis for the column space $\mathbf{CS}(A)$ of A and a basis for the nullspace $\mathbf{NS}(A)$. Verify that $\dim(\mathbf{NS}(A)) = n - \operatorname{rank}(A)$.

Exercise 7 (14 pts)**.** We denote by $\mathbf{P}_i$ the set of all polynomials of degree less than or equal to i. Let $T_1 : \mathbf{P}_1 \longrightarrow \mathbf{P}_2$ be the linear transformation defined by $T_1(p(x)) = xp(x)$ and let $T_2 : \mathbf{P}_2 \longrightarrow \mathbf{P}_2$ be the linear operator defined by $T_2(p(x)) = p(2x+1)$. Let $\mathcal{B} = \{1, x\}$ and $\mathcal{B}' = \{1, x, x^2\}$ be the standard bases for $\mathbf{P}_1$ and $\mathbf{P}_2$, respectively.

(1) Find the matrix of T_1 with respect to the bases $\mathcal{B}$ and $\mathcal{B}'$.

(2) Find the matrix of T_2 with respect to the basis $\mathcal{B}'$.

(3) Find the matrix of $T_2 \circ T_1$ with respect to the bases $\mathcal{B}$ and $\mathcal{B}'$.

Exercise 8 (12 pts)**.** Let $\mathcal{B} = (\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3)$ be a basis for a vector space V, and let $T : V \longrightarrow V$ be the linear transformation with matrix relatively to $\mathcal{B}$:

$$M = \begin{pmatrix} -3 & 4 & 7 \\ 1 & 0 & -2 \\ 0 & 1 & 0 \end{pmatrix}$$

(1) Set $\mathbf{v}_1 = \mathbf{u}_1$, $\mathbf{v}_2 = \mathbf{u}_1 + \mathbf{u}_2$ and $\mathbf{v}_3 = \mathbf{u}_1 + \mathbf{u}_2 + \mathbf{u}_3$. Show that $\mathcal{B}' = (\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3)$ is a basis of V.

(2) Find the matrix of T with respect to $\mathcal{B}'$.