KFUPM, DEPARTMENT OF MATHEMATICS AND STATISTICS

MATH 280: TEST 2, SEMESTER (132), MARCH 17, 2014

Time: 18:30 to 20:00

Name : ....

ID : .... Section : ....

Exercise	**Points**
1	____ / 15
2	____ / 12
3	____ / 15
4	____ / 8
Total	____ / 50

Exercise 1. Let a, b, c, d be complex numbers. Evaluate the following determinants:

(i) $V(a,b) = \begin{vmatrix} 1 & a \\ 1 & b \end{vmatrix}$

(ii) $V(a,b,c) = \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$

(iii) $V(a,b,c,d) = \begin{vmatrix} 1 & a & a^2 & a^3 \\ 1 & b & b^2 & b^3 \\ 1 & c & c^2 & c^3 \\ 1 & d & d^2 & d^3 \end{vmatrix}$

Exercise 2. Let A be a nonsingular $n \times n$-matrix with $n > 1$.

(a) Show that
$$\det(\operatorname{adj}(A)) = (\det(A))^{n-1}.$$

(b) Show that $\operatorname{adj}(A)$ is nonsingular and
$$(\operatorname{adj}(A))^{-1} = \det(A^{-1})A = \operatorname{adj}(A^{-1}).$$

(c) Show that a square matrix M is singular if and only if $\operatorname{adj}(M)$ is also singular.

Exercise 3. Let A be a $k \times k$-matrix and B be an $(n-k) \times (n-k)$-matrix.

Consider the matrices $E = \begin{pmatrix} I_k & O \\ & \\ O & B \end{pmatrix}$, $F = \begin{pmatrix} A & O \\ & \\ O & I_{n-k} \end{pmatrix}$ and $C = \begin{pmatrix} A & O \\ & \\ O & B \end{pmatrix}$.

(a) Show that $\det(E) = \det(B)$.

(b) Show that $\det(F) = \det(A)$.

(c) Show that $\det(C) = \det(A)\det(B)$.

(d) Let $M = \begin{pmatrix} O & B \\ & \\ A & O \end{pmatrix}$. Show that $\det(M) = (-1)^k \det(A)\det(B)$.

Exercise 4. Given the matrix

$$A \begin{pmatrix} 1 & 2 & 0 \\ k & 1 & k \\ 0 & 2 & 1 \end{pmatrix}.$$

(a) Evaluate $\det(A)$.

(b) Find the values of k for which the above matrix A is invertible.