KFUPM, DEPARTMENT OF MATHEMATICS AND STATISTICS

MATH 280: TEST 1, SEMESTER (132), FEBRUARY 18, 2014

Time: 18:40 to 20:00

Name : ....

ID : .... Section : ....

Exercise	**Points**
1	____ 12
2	____ 6
3	____ 12
4	____ 20
Total	____ 50

Exercise 1. Consider the linear system whose augmented matrix is given by

$$\left(\begin{array}{ccc:c} 1 & 1 & 3 & 2 \\ 1 & 2 & 4 & 3 \\ 1 & 3 & a & b \end{array}\right)$$

(**a**) For what values of a and b will the system have infinitely many solutions?

(**b**) For what values of a and b will the system be inconsistent?

Exercise 2. Let A be an $m \times n$ matrix such that $n \geq 3$.

Consider the system $AX = B$. Suppose that

$$B = \mathrm{col}_2(A) - \mathrm{col}_3(A) = \mathrm{col}_1(A) + 2\mathrm{col}_2(A).$$

Show that the system has infinitely many solutions.

Exercise 3. Let A be an $n \times n$-matrix such that $A^2 = A$.
Consider the matrix $M = A + 2I_n$.

(**a**) Show that $M^2 - 5M + 6I_n = \mathbf{O}$.

(**b**) Deduce from (a) that M is nonsingular and find M^{-1} (as a function of M).

Exercise 4. Let A be the matrix given by

$$\begin{pmatrix} 1 & 0 & 1 \\ 3 & 3 & 4 \\ 2 & 2 & 3 \end{pmatrix}$$

(**a**) Find the inverse of A by reducing the augmented matrix $[A \vdots I_3]$

(**b**) Express A as a product of elementary matrices.

(**c**) Solve the system

$$\begin{cases} x & & & + & z & = & 2 \\ 3x & + & 3y & + & 4z & = & 1 \\ 2x & + & 2y & + & 3z & = & 1 \end{cases}$$

(**d**) Find an LU-factorization of A.