King Fahd University of Petroleum and Minerals

Department of Mathematics and Statistics

Math260 (Introduction to Differential Equations and Linear Algebra)

Exam II (Term 132)

Wednesday, April 16, 2014 Time Allowed: 2 hours

Name: ID Number
Section Number: Serial Number:
Class Time: Instructor's Name:

Instruction:

1. Calculators and Mobiles are not allowed.
2. Write neatly and legibly. You may lose points for messy work.
3. Show all your work. No points for answers without justification.
4. Make sure that you have 6 pages of problems (Total of 9 problems)

Question #	Points	Max. Points
1		10
2		10
3		6
4		16
5		10
6		15
7		8
8		10
9		15
Total		100

Q.1. Solve the system

$$
\begin{aligned}
&x_1 + 6x_2 + 2x_3 - 5x_4 - 2x_5 = -4 \\
&2x_3 - 8x_4 - x_5 = 3 \\
&2x_5 = 14
\end{aligned}
$$

by transforming its augmented matrix into reduced echelon form.

Q.2. Let

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

a) Find a 2×2 matrix B such that $\det(A+B) \neq \det A + \det B$

b) Find a 2×2 nonzero matrix C such that $\det(A+C) = \det A + \det C$

Q.3. The vectors $\mathrm{u} = (1,1,0)$, $\mathrm{v} = (4,3,1)$, $\mathrm{w} = (0,1,\mathrm{c})$ are linearly dependent. Find c .

Q.4. Which of the following is a subspace of $\mathbb{R}^4$?

a) W is set of all vectors $(x_1, x_{2,} x_3, x_4) \in \mathbb{R}^4$ such that $x_1 + x_3 = x_2 + x_4$.

b) W is set of all vectors $(x_1, x_{2,} x_3, x_4) \in \mathbb{R}^4$ such that $x_1 x_2 = x_3 + x_4$.

Q.5. Express the vector $\mathrm{t} = (-2, -2, 2, 2)$ as a linear combination of the vectors $\mathrm{u} = (1, 2, 1, 2), \mathrm{v} = (1, 2, 1, 0),\ \mathrm{w} = (0, 1, 2, 0)$.

Q.6.

a) Verify that the solutions $y_1 = x, y_2 = x^2, y_3 = x^{-4}$ of the DE

$x^3 y''' + 4x^2 y'' - 8xy' + 8y = 0$ are linearly independent on $(0, \infty)$.

b) Find the solution of the IVP:

$x^3 y''' + 4x^2 y'' - 8xy' + 8y = 0; y(1) = 1, y'(1) = 1, y''(1) = 10$

Q.7. Find a differential equation of order 2 having the general solution

$$y = c_1 + c_2 e^{-3x}$$

Q.8. Find a basis of the subspace of $\mathbb{R}^5$ consisting of all vectors $(x,\ y, -y,\ x - y,\ z)$.

Q.9.

Find the inverse of the coefficient matrix and hence the solution of the following system

$$4x_1 + 6x_2 - 3x_3 = 0$$
$$2x_1 + 3x_2 - 4x_3 = 0$$
$$x_1 - x_2 + 3x_3 = -7$$