King Fahd University of Petroleum and Minerals Department of Mathematics and Statistics Math 202 Section 07 Quiz I (Term 132)

1. State the order of the given ordinary differential equation. Determine whether the equation is linear or nonlinear. (State why?)

a)
$$x\frac{d^4y}{dx^3} - \left(\frac{dy}{dx}\right)^5 + y = 0$$

b)
$$(\sin \theta) y''' - (\cos \theta) y' y = 2$$

2. Determine the value of m for which the differential equation y''' - 4y'' + 3y' = 0 has solutions of the form $y = e^{mx}$.

3. Verify that $y \sin x + x^2 e^y - y = c$ is an implicit solution for the differential equation

$$(y\cos x + 2xe^y) + (\sin x + x^2e^y - 1)y' = 0.$$

4. Consider the differential equation $(y^2 - y) + \frac{dy}{dx} = 0$

a) Verify that $y = \frac{e^x}{c + e^x}$ is a one-parameter family of solutions of the given differential equation.

b) Find two constant solutions of the given differential equation.

c) Find a singular soltion of the given differential equation.

5. a) Determine a region in which the differential equation $y' = \sqrt{\frac{y^2 - 9}{x}}$ has a unique solution through the point (x_0, y_0)

b) Determine whether the existence of a unique solution Theorem guarantees that the differential equation $y' = \sqrt{\frac{y^2 - 9}{x}}$ has a unique solution through the point (1, 3)

6. a) Find the value of C such that $y = \frac{1}{x^2 + C}$ is a solution of the initial value problem $y' + 2xy^2 = 0, y(0) = -1.$

b) Give the largest interval over which the solution is defined.

7. Solve

$$x^{2}\frac{dy}{dx} = y - xy$$
$$y(-1) = -1$$