Quiz 2

Name:

ID#:

Q_1. Find $I = \int (\sinh x + \cosh x)^3 \cdot \cosh x \, dx$

- $\sinh x + \cosh x = \dfrac{e^x - e^{-x}}{2} + \dfrac{e^x + e^{-x}}{2} = e^x$

- $I = \int (e^x)^3 \cdot \dfrac{e^x + e^{-x}}{2} \, dx$

$= \frac{1}{2} \int (e^{4x} + e^{2x}) \, dx = \frac{1}{2} \left[\frac{1}{4} e^{4x} + \frac{1}{2} e^{2x} \right] + C$

Q_2. Find $J = \int \cot x . \sqrt{\csc x}\, dx$

Let $u = \csc x$

then: $du = -\csc x . \cot x\, dx$

$J = \int \cot x . \csc x . \csc^{-\frac{1}{2}} x\, dx$

$= -\int u^{-\frac{1}{2}}\, du = -2 u^{\frac{1}{2}} + C$

$= -2 \csc^{\frac{1}{2}} x + C.$

Q_3. Find $K = \int \sin 2x \cdot \sin 4x \, dx$

$$\sin 2x \cdot \sin 4x = \frac{1}{2}\left[\cos(2x - 4x) - \cos(2x + 4x)\right]$$

$$= \frac{1}{2}\left[\cos 2x - \cos 6x\right] \qquad (\cos -2x = \cos 2x)$$

$$K = \frac{1}{2}\int \left[\cos 2x - \cos 6x\right] dx$$

$$= \frac{1}{2}\left[\frac{1}{2}\sin 2x - \frac{1}{6}\sin 6x\right] + c$$

Q4. Find the reduction formula for:

$$I_n = \int (1+x^2)^n \, dx$$

$$u = (1+x^2)^n \qquad dv = dx$$

$$du = n(1+x^2)^{n-1}(2x)\,dx \qquad v = x$$

By integration by part: $I_n = x(1+x^2)^n - 2n \underbrace{\int x^2 (1+x^2)^{n-1} \, dx}_{M}$

$$M = \int x^2 (1+x^2)^{n-1} \, dx = \int (x^2+1-1)(1+x^2)^{n-1} \, dx$$

$$= \int \left[(x^2+1)^n - (x^2+1)^{n-1} \right] dx$$

$$= I_n - I_{n-1}$$

So $I_n = x(1+x^2)^n - 2n\left[I_n - I_{n-1}\right]$

$$\Rightarrow \quad 2n I_n + I_n = x(1+x^2)^n + 2n I_{n-1}$$

$$\Rightarrow \quad I_n = \frac{1}{2n+1}\left[x(1+x^2)^n + 2n I_{n-1}\right] \qquad \square$$