

KEY

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(show all your work)

1) (DONOT EVALUATE THE INTEGRALS). Write the suitable u and dv if we use the integration by parts.

2 each

integral	u	dv
$\int x \sin x dx$	$u = x$	$dv = \sin x dx$
$\int \ln(3x) dx$	$u = \ln(3x)$	$dv = dx$
$\int x \tan^{-1} x dx$	$u = \tan^{-1} x$	$dv = x dx$
$\int \ln(\cos x) dx$	$u = \ln(\cos x)$	$dv = dx$
$\int x (\ln x)^2 dx$	$u = (\ln x)^2$	$dv = x dx$
$\int x \sec^2 x dx$	$u = x$	$dv = \sec^2 x dx$
$\int x \sec x \tan x dx$	$u = x$	$dv = \sec x \tan x dx$

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2) Evaluate the integrals $\int e^{-5x} \sin(\frac{3}{2}x) dx = I$ two integrations by parts

$$I = e^{-5x} \left[-\frac{1}{5} \sin(\frac{3}{2}x) - \frac{3}{50} \cos(\frac{3}{2}x) \right] - \frac{9}{100} \int e^{-5x} \sin(\frac{3}{2}x) dx$$

$\begin{matrix} \sin(\frac{3}{2}x) & \xrightarrow{+} & e^{-5x} \\ & \searrow & \\ \frac{3}{2} \cos(\frac{3}{2}x) & \xrightarrow{-} & -\frac{1}{5} e^{-5x} \\ & \searrow & \\ -\frac{9}{4} \sin(\frac{3}{2}x) & \xrightarrow{+} & \frac{1}{25} e^{-5x} \end{matrix}$

$$\Rightarrow I = e^{-5x} \left[-\frac{1}{5} \sin(\frac{3}{2}x) - \frac{3}{50} \cos(\frac{3}{2}x) \right] - \frac{9}{100} I$$

$$\left(1 + \frac{9}{100}\right) I = e^{-5x} \left[-\frac{1}{5} \sin(\frac{3}{2}x) - \frac{3}{50} \cos(\frac{3}{2}x) \right]$$

$$I = \frac{100}{109} e^{-5x} \left[-\frac{1}{5} \sin(\frac{3}{2}x) - \frac{3}{50} \cos(\frac{3}{2}x) \right] + C$$

$$\frac{e^{-5x}}{109} \left[-20 \sin(\frac{3}{2}x) - 6 \cos(\frac{3}{2}x) \right] + C$$

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$$e^{-5x} \sin\left(\frac{3}{4}x\right) \cos\left(\frac{3}{4}x\right)$$

$$\frac{1}{2} e^{-5x} \sin\left(\frac{3}{2}x\right)$$

$$\sin(2\theta) = 2\sin\theta\cos\theta$$

$$I = \frac{1}{2} \int e^{-5x} \sin\left(\frac{3}{2}x\right) dx$$

$$= \frac{1}{2} e^{-5x} \left[-\frac{1}{5} \sin\left(\frac{3}{2}x\right) - \frac{3}{50} \cos\left(\frac{3}{2}x\right) \right]$$

$$- \frac{9}{100} I \quad \triangle 3$$

$$\left(1 + \frac{9}{100} I\right) = \frac{1}{2} e^{-5x} \left[-\frac{1}{5} \sin\left(\frac{3}{2}x\right) - \frac{3}{50} \cos\left(\frac{3}{2}x\right) \right]$$

$$I = \left(\frac{100}{109}\right) \left(\frac{1}{2}\right) e^{-5x} \left[-\frac{1}{5} \sin\left(\frac{3}{2}x\right) - \frac{3}{50} \cos\left(\frac{3}{2}x\right) \right] + C$$

$$= \frac{1}{109} e^{-5x} \left[-10 \sin\left(\frac{3}{2}x\right) - 3 \cos\left(\frac{3}{2}x\right) \right] + C$$

$\triangle 3$

$$\triangle 6$$

$$\sin\left(\frac{3}{2}x\right) \quad e^{-5x}$$

$$\frac{3}{2} \cos\left(\frac{3}{2}x\right) \quad + \quad -\frac{1}{5} e^{-5x}$$

$$-\frac{9}{4} \sin\left(\frac{3}{2}x\right) \quad - \quad \frac{1}{25} e^{-5x}$$