

(show all your work)

1) If $\tanh x = -\frac{3}{4}$ then find $\sinh x$

$$\operatorname{sech}^2 x = 1 - \tanh^2 x = 1 - \frac{9}{16} = \frac{7}{16}$$

$$\Rightarrow \operatorname{sech} x = \sqrt{7}/4 \quad (\text{Note that } \cosh \text{ always positive})$$

$$\Rightarrow \cosh x = 4/\sqrt{7}$$

$$\text{Now, } \cosh^2 x - \sinh^2 x = 1 \Rightarrow \cosh^2 x = 1 + \sinh^2 x$$

~~$$\cosh^2 x = 1 + \sinh^2 x$$~~ or $\sinh^2 x = -1 + \cosh^2 x$

$$\sinh^2 x = -1 + \frac{16}{7} = \frac{9}{7} \Rightarrow \sinh^2 x = \frac{-3}{\sqrt{7}} \quad \triangle 10$$

Note that: $\tanh x$ is negative $\Rightarrow \sinh x$ is negative

$$\begin{aligned} \cosh^2 x - \sinh^2 x &= 1 \\ 1 - \tanh^2 x &= \operatorname{sech}^2 x \end{aligned}$$

2) Find the area of the region enclosed by the curves $y = \cosh x$, $y = e^x$ and $x = 1$

$$\text{Area} = \int_0^1 (e^x - \cosh x) dx \quad \triangle 3$$

$$= [e^x - \sinh x]_0^1 \quad \triangle 3$$

$$= [e - \sinh(1)] - [1 - \sinh(0)]$$

$$= [e - \sinh(1)] - [1 - 0]$$

$$= e - \sinh(1) - 1 \quad \triangle 2$$

