Name:	**ID:**	**Sec**: **34**(11:00-11:50) **23**(1:10-2:00)

MATH-102 **Term-132** **Long Class-QUIZ-5 FormA**

(circle one letter only)

1)The average value of the function $f(x)=\frac{\tan x}{x^2+1}+\sqrt{4-x^2}$ over the interval [-2,2] is

$f_{avg}=\frac{1}{b-a}\int_a^b f(x)dx$

$=\frac{1}{4}\int_{-2}^{2}\left(\frac{\tan x}{x^2+1}+\sqrt{4-x^2}\right)dx$

$=\frac{1}{4}\left[\int_{-2}^{2}\text{odd}+\int_{-2}^{2}\sqrt{4-x^2}dx\right]$

$=\frac{1}{4}[0+\text{half circle}]$

$=\frac{1}{4}\left[\frac{1}{2}(4\pi)\right]$

$=\pi/2$

(a) 0

(b) π/4

(c) π/3

(d) $\pi/2$

(e) π

(f) None of the above

2) $\int_0^1 \frac{e^{\tan^{-1}x}}{1+x^2}dx=$

$u=\tan^{-1}x \rightarrow du=\frac{dx}{1+x^2}$

$=\int_0^{\pi/4} e^u du=\left[e^u\right]_0^{\pi/4}$

$=e^{\pi/4}-1$

(a) $1-e^{\pi/2}$

(b) $e^{\pi}-\pi$

(c) $e^{\pi/2}-1$

(d) $e^{-\pi/4}-2$

(e) $e^{\pi/4}-1$

(f) None of the above

3) $\int_0^1(1+x)\sqrt{4-4x}dx=$

$\int_0^1(1+x)\sqrt{4}\sqrt{1-x}\,dx$

$=2\int_0^1(1+x)\sqrt{1-x}\,dx$

Let $u=1-x \rightarrow du=-dx$

$=-2\int_1^0(2-u)\sqrt{u}\,du$

$=-2\int_1^0\left(2u^{1/2}-u^{3/2}\right)du$

$=-2\left[\frac{4}{3}u^{3/2}-\frac{2}{5}u^{5/2}\right]_1^0$

$=-2\left[(0)-\left(\frac{4}{3}-\frac{2}{5}\right)\right]$

$=2\left(\frac{14}{15}\right)=28/15$

(a) 28/15

(b) 15/28

(c) 14/15

(d) 28/13

(e) 15/13

(f) None of the above

4) If g is a continuous function such that $\int_0^{2x}e^{t/2}g(t)dt=\frac{1}{2}xe^x$, then g(4) =

diff w.r.t x both sides

$2e^x g(2x)=\frac{1}{2}(e^x+xe^x)$

$\Rightarrow g(2x)=\frac{1}{4}(1+x)$

let $x=2$

$g(4)=3/4$

(a) 4/3

(b) 2/3

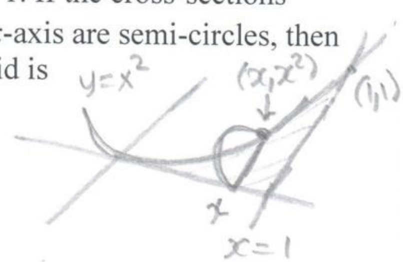
(c) 3/2

(d) 4/5

(e) 3/4

(f) None of the above

5) The base of a solid is bounded by the curves $y=x^2$, $y=0$ and $x=1$. If the cross-sections perpendicular to the x-axis are semi-circles, then the volume of the solid is



(a) π/4

(b) 40/π

(c) π/40

(d) 4/π

(e) 2π

(f) None of the above

diameter $=x^2$

radius $=\frac{1}{2}x^2$

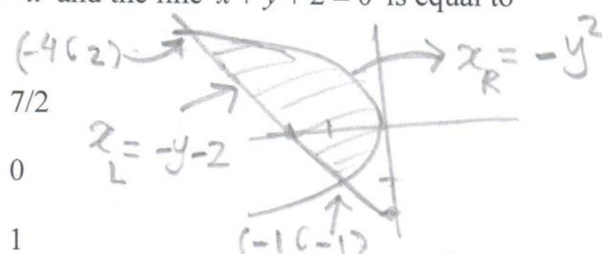
Area of cross section is

$\frac{1}{2}\pi\left(\frac{1}{2}x^2\right)^2=\frac{\pi}{8}x^4$

$V=\int_0^1\frac{\pi}{8}x^4dx=\frac{\pi}{8}\left[\frac{1}{5}x^5\right]_0^1$

$=\pi/40$

6) The area of the region enclosed by the curves $y^2=-x$ and the line $x+y+2=0$ is equal to



(a) 7/2

(b) 0

(c) 1

(d) 9/2

(e) 5/2

(f) None of the above

intersection: $y^2=y+2$

$\rightarrow y^2-y-2=0 \rightarrow$

$(y-2)(y+1)=0$

$A=\int_{-1}^{2}(x_R-x_L)dy=\int_{-1}^{2}(-y^2+y+2)dy$

$=\left[-\frac{1}{3}y^3+\frac{1}{2}y^2+2y\right]_{-1}^{2}=\left(-\frac{8}{3}+6\right)-\left(\frac{1}{3}+\frac{1}{2}-2\right)$

$=-3+8-\frac{1}{2}=9/2$

(circle one letter)

7) Let P be a partition of the interval [0, 2], then the limit $\lim_{\|P\|\to 0} \sum_{k=1}^{n} \left[c_k + c_k^2\right]\Delta x_k =$ $R_n = \sum_{k=1}^{n} \Delta x_k f(x_k)$

$= \int_0^2 (x + x^2)\,dx$

$= \left[\frac{1}{2}x^2 + \frac{1}{3}x^3\right]_0^2$

$= (2 + \frac{8}{3}) - (0)$

$= 14/3$

(a) 12/3

(b) 14/3

(c) 16/3

(d) 18/3

(e) 20/3

(f) None of the above

8) The region in the first quadrant enclosed by the parabolas $y = x^2$, $y = 2 - x^2$, and the y-axis is rotating about the line $x = -1$, then the volume of the solid generated is given by

(a) $\int_0^1 \pi(1 - x^4)dx$

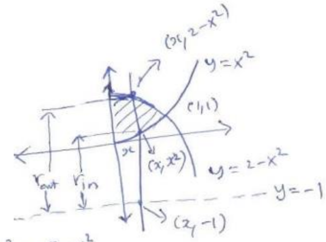
(b) $\int_0^2 8\pi(1 + x^2)dx$

(c) $\int_0^1 4\pi(1 - x^2)dx$

(d) $\int_0^1 8\pi(1 - x^2)dx$

(e) $\int_0^1 4\pi(1 + x^2)dx$

(f) None of the above



$r_{out} = 2 - x^2 + 1 = 3 - x^2$

$r_{in} = x^2 + 1$

$A = \pi\left((3 - x^2)^2 - (x^2+1)^2\right)$

$= \pi(9 - 6x^2 + x^4 - x^4 - 2x^2 - 1)$

$= \pi(8 - 8x^2)$

$V = \int_0^1 \pi(8 - 8x^2)dx$

9) If f is an even function such that $\int_{-3}^{3} f(t)dt = 10$ and $\int_{-2}^{2} f(t)dt = 16$, then $\int_{-3}^{-2} f(t)dt =$

(a) -3

(b)

(c)

(d)

(e)

(f) None of the above

$\int_{-3}^{3} = 10 \Rightarrow \int_{-3}^{0} = 5$

$\int_{-2}^{2} = 16 \Rightarrow \int_{-2}^{0} = 8$

$\int_{-3}^{0} = \int_{-3}^{-2} + \int_{-2}^{0}$

$5 = \int_{-3}^{-2} + 8$

$\Rightarrow \int_{-3}^{-2} = -3$

10) If $F(x) = \int_{1/2}^{2x} f(t)dt$ and $f(t) = \int_{1/2}^{t^2} \frac{\sqrt{1+u^2}}{u}dt$ then $F''(1) =$

(a) $5\sqrt{17}$

(b) $4\sqrt{17}$

(c) $3\sqrt{17}$

(d) $2\sqrt{17}$

(e) $\sqrt{17}$

(f) None of the above

$F'(x) = 2f(2x)$

Now, $f'(t) = \frac{\sqrt{1+t^4}}{t^2}(2t)$

$\Rightarrow F''(x) = (2)f'(2x)(2)$

$F''(x) = 4f'(2x)$

$\Rightarrow F''(1) = 4f'(2) =$

$= 4\left(\frac{\sqrt{17}}{4}\right)(4) = 4\sqrt{17}$

11) $\int_0^{\frac{\pi}{4}} (\sec x + \cos x)^2\,dx =$ $\int_0^{\pi/4} (\sec^2 x + 2 + \cos^2 x)$

$= \left[\tan x + 2x\right]_0^{\pi/4} + \int_0^{\pi/4} \cos^2 x\,dx$

$= \left[1 + \frac{\pi}{2}\right] + \int_0^{\pi/4} (\frac{1}{2} + \frac{1}{2}\cos 2x)$

$= \left[1 + \frac{\pi}{2}\right] + \left[\frac{1}{2}x + \frac{1}{4}\sin 2x\right]_0^{\pi/4}$

$= 1 + \frac{\pi}{2} + \frac{\pi}{8} + \frac{1}{4}$

$= \frac{10}{8} + \frac{5\pi}{8}$

(a) (5π+10)/8

(b) (4π+10)/8

(c) (3π+10)/8

(d) (2π+10)/8

(e) (π+10)/8

(f) None of the above

12) $I = \int \frac{\ln(\tan^{-1} x)}{(x^2+1)\tan^{-1} x}dx =$

let $u = \ln(\tan^{-1} x)$

$du = \frac{dx}{(1+x^2)\tan^{-1} x}$

$I = \int u\,du$

$= \frac{1}{2}u^2 + C$

$= \frac{1}{2}\left[\ln(\tan^{-1} x)\right]^2 + C$

(a) $\left[\ln(\tan^{-1} x)\right]^2 + C$

(b) $\frac{1}{2}\left[\ln(\tan^{-1} x)\right] + C$

(c) $\frac{1}{2}\left[\ln(\tan^{-1} x)\right]^2 + C$

(d) $\frac{1}{2}\left[\tan^{-1} x\right]^2 + C$

(e) $\ln(\tan^{-1} x) + C$

(f) None of the above