Name: KEY	ID: KEY	Sec: 34 (11:00-11:50) 23 (1:10-2:00)

MATH-102 Term-132 CQ-15 **FORM A**

(show your work and circle one letter to get a full mark or you will get zero)

1) The series $\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{(n+1)^2+3^n}$ is

(a) convergent by the integral test.
(b) conditionally convergent.
(c) divergent.
(d) divergent by the alternating series test.
(e) absolutely convergent.
(f) none of the above

First we study $\sum |a_n|$

$(n+1)^2+3^n > 3^n$

$\Rightarrow \frac{1}{(n+1)^2+3^n} < \frac{1}{3^n}$ ← geometric with $r=1/3$ (convg)

by comparison test $\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{(n+1)^2+3^n}$ is AC

2) The series $\sum_{n=2}^{\infty}(-1)^n\frac{1}{n(\ln n)^2}$ is

(a) absolutely convergent.
(b) conditionally convergent.
(c) convergent as its sum is zero.
(d) divergent by the alternating series test.
(e) convergent as its sum is $\ln 2$
(f) none of the above

First we study $\sum |a_n| = \sum_{n=2}^{\infty}\frac{1}{n(\ln n)^2}$

$\int_2^{\infty}\frac{dx}{x(\ln x)^2} = \lim_{t\to\infty}\left[-\frac{1}{\ln x}\right]_2^t = \lim_{t\to\infty}\left[-\frac{1}{\ln t}+\frac{1}{\ln 2}\right]$

$= 0 + \frac{1}{\ln 2} \Rightarrow \sum a_n$ is AC

3) The series $\sum_{n=1}^{\infty}\frac{1}{\sqrt{n}(\sqrt{1+n}+1)}$ is

(a) converges by the integral test
(b) diverges by the ratio test.
(c) converges by the ratio test.
(d) converges by the root test.
(e) diverges by the limit comparison test.
(f) none of the above

we use limit comparison with

$\frac{1}{\sqrt{n}\sqrt{1+n}} = \frac{1}{\sqrt{n^2+n}} \approx \frac{1}{\sqrt{n^2}} = \frac{1}{n}$

$c = \lim_{n\to\infty}\frac{1/n}{1/\sqrt{n}(\sqrt{1+n}+1)} = \lim_{n\to\infty}\frac{\sqrt{n}(\sqrt{n+1}+1)}{n}$

$= \lim_{n\to\infty}\frac{\sqrt{n+1}+1}{\sqrt{n}} = 1 \Rightarrow$ both divg

4) The series $\sum_{n=1}^{\infty}\frac{(-1)^n(n!)^4}{(4n+3)!}$ is

(a) a divergent p series.
(b) conditionally convergent
(c) divergent by the ratio test.
(d) absolutely convergent.
(e) a series for which the Ratio test is inconclusive.
(f) none of the above

First $\sum |a_n|$ by ratio test

$\left|\frac{a_{n+1}}{a_n}\right| = \frac{[(n+1)!]^4}{(4n+7)!}\cdot\frac{(4n+3)!}{(n!)^4}$

$(4n+7)(4n+6)(4n+5)(4n+4)$

$= \frac{(n+1)^4}{(4n+7)(4n+6)(4n+5)(4n+4)} \to \frac{1}{4^4} < 1$

Hence AC

5) The series $\sum_{n=1}^{\infty}\frac{3^n n^n}{2^{2n+1}}$ is

(a) a convergent p series.
(b) converges by the root test.
(c) a series for which the root test is inconclusive
(d) a divergent geometric series.
(e) diverges by the root test.
(f) none of the above

$a_n = \frac{3^n n^n}{2^{2n}\cdot 2} = \frac{3^n n^n}{4^n\cdot 2} = \left(\frac{3n}{4}\right)^n\frac{1}{2}$

$\sqrt[n]{a_n} = (a_n)^{1/n} = \left(\frac{3n}{4}\right)\left(\frac{1}{2}\right)^{1/n} \to (+\infty)(1) = \infty$

$\infty > 1 \Rightarrow$ divs by root

$\lim_{n\to\infty} -a_n = \infty$ (by two $L'H$) $y = \cosh x$

6) The series $\sum_{n=2}^{\infty}(-1)^n\left(\sqrt{n+3}-\sqrt{n+2}\right)$ is

(a) diverges by the limit comparison test
(b) converges conditionally
(c) converges absolutely
(d) diverges by the Ratio Test
(e) diverges by the nth-term test for divergence.
(f) none of the above

$= \sum \frac{(-1)^n}{\sqrt{n+3}+\sqrt{n+2}}$

First multiply by conjugate $(\sqrt{n+3}-\sqrt{n+2}) = \frac{1}{\sqrt{n+3}+\sqrt{n+2}}$

we study $\sum_{n=2}^{\infty} |a_n| = \sum_{n=2}^{\infty} \frac{1}{\sqrt{n+3}+\sqrt{n+2}}$

limit comparison $\sum \frac{1}{\sqrt{n}}$, $\lim_{n\to\infty} \frac{\sqrt{n}}{\sqrt{n+3}+\sqrt{n+2}} = \lim_{n\to\infty} \frac{\sqrt{1}}{\sqrt{1+\frac{3}{n}}+\sqrt{1+\frac{2}{n}}} = 2$

both div. by Alt. ser. Test ① alternating

② $\lim_{n\to\infty} u_n = 0$ ③ decreasing $f(x) = \frac{1}{\sqrt{x+3}+\sqrt{x+2}} \Rightarrow CC$

7) The series $\sum_{n=1}^{\infty}\frac{(2-\cosh n)}{n\sqrt{n}}$ is

(a) divergent by ratio test.
(b) diverges by the divergence test.
(c) convergent by comparison test.
(d) divergent by the integral test.
(e) convergent by the ratio test.
(f) none of the above

$1 \leq \cosh n$

$-1 \geq -\cosh n$

$2-1 \geq 2-\cosh n$

$\frac{1}{n\sqrt{n}} \geq \frac{2-\cosh n}{n\sqrt{n}}$

$\frac{2-\cosh n}{n\sqrt{n}}$ negative terms

8) The series $\sum_{n=1}^{\infty}(3-\sqrt[n]{3})^{\frac{n}{2}}$ is

positive terms

(a) converges by root test.
(b) diverges by the root test.
(c) the root test is inconclusive.
(d) a divergent geometric series
(e) converges by using the comparison test.
(f) none of the above

$(a_n)^{1/n} = (3-3^{1/n})^{1/2} \to (3-1)^{1/2} = \sqrt{2} > 1$

by root div

9) By applying **the ratio test** to the series $\sum_{n=0}^{\infty}\frac{1}{(\ln n)^{|p|+1}}$ we conclude that

(a) the series is absolutely convergent
(b) the series is convergent
(c) the series is divergent
(d) the test is inconclusive.
(e) the series is conditionally convergent
(f) none of the above

$\frac{a_{n+1}}{a_n} = \frac{[\ln(n+1)]^{|P|+1}}{[\ln(n)]^{|P|+1}} = \left[\frac{\ln(n+1)}{\ln(n)}\right]^{|P|+1}$

$\lim_{n\to\infty} \frac{\ln(n+1)}{\ln n} \overset{L'H}{=} \lim_{n\to\infty} \frac{1/(n+1)}{1/n} = 1$

$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \left[\lim_{n\to\infty} \frac{\ln(n+1)}{\ln(n)}\right]^{|P|+1} = 1^{|P|+1} = 1$

10) The series $\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{\sqrt{n+4}}$ is

(a) divergent
(b) absolutely convergent.
(c) conditionally convergent.
(d) neither convergent nor divergent.
(e) a convergent p-series
(f) none of the above

First, we study $\sum |a_n|$ by limit comp.

$\lim_{n\to\infty} \frac{1/\sqrt{n}}{1/\sqrt{n+4}} = 1 \Rightarrow \sum |a_n|$ divg.

Now, let us use Alt. ser. test

① it is alternating

② $u_n = \frac{1}{\sqrt{n+4}} \to 0$ as $n\to\infty$

③ $f(x) = \frac{1}{\sqrt{x+4}} = (x+4)^{-1/2} \Rightarrow f'(x) = -\frac{1}{2}(x+4)^{-3/2}$

$f'(x) = \frac{-1}{2(x+4)^{3/2}} < 0$ for all $x \geq 1$

(i.e) decreasing series

Hence CC