Name: KEY	ID: KEY	Sec: 34(11:00-11:50) 23(1:10-2:00)

MATH-102 Term-132 CQ-14

(show all your work and circle one letter to get a full mark or you will get zero)

1) If $\{S_n\}$ is the sequence of partial sums of the series $\sum_{n=1}^{\infty}\frac{1}{(n+1)(n+2)}$, then $S_{98}=$

(a) 0.25
(b) 0.15
(c) 0.6
(d) 0.499
(e) 0.76
(f) 0.92
(g) 0.49
(f) none of the above

$$\frac{1}{(n+1)(n+2)} = \frac{1}{n+1} - \frac{1}{n+2}$$

telescoping

$$S_n = \frac{1}{2} - \frac{1}{n+2}$$

$$S_{98} = \frac{1}{2} - \frac{1}{100}$$

$$= 0.5 - 0.01$$

$$= 0.49$$

2) $\frac{1}{4}-\frac{1}{8}+\frac{1}{16}-\frac{1}{32}+\frac{1}{64}-\frac{1}{128}+\frac{1}{256}=$ z ~~$2(-\frac{1}{2})$~~

(a) $\frac{1}{6}(\frac{255}{256})$
(b) $\frac{255}{256}$
(c) $\frac{39}{54}$
(d) $\frac{86}{13}$
(e) $\frac{1}{3}(\frac{59}{54})$
(f) none of the above

$$z = \frac{1}{4}\left(1-\frac{1}{2}+\frac{1}{4}-\frac{1}{8}+\frac{1}{16}-\frac{1}{32}+\frac{1}{64}\right)$$

$$= \frac{1}{4}\left(\sum_{n=1}^{7}\left(-\frac{1}{2}\right)^{n-1}\right)$$

geom with $r=-1/2$, $a=1$

n-th partial sum $= S_n = a\frac{1-r^n}{1-r}$

$$S_n = \frac{1-(-\frac{1}{2})^n}{1-(-\frac{1}{2})} \Rightarrow S_7 = \frac{1+\frac{1}{2^7}}{1+\frac{1}{2}} = \frac{2^7+1}{2^7+2^6}$$

$$= \frac{2^7+1}{2^6(2+1)} = \frac{128+1}{(64)(3)}$$

So, $z = \frac{1}{4}\left(\frac{129}{(64)(3)}\right) = \boxed{\frac{1}{6}\left(\frac{129}{128}\right)}$

Another sol:

$$z = \frac{1}{4}-\frac{1}{8}+\frac{1}{16}-\frac{1}{32}+\frac{1}{64}-\frac{1}{128}+\frac{1}{256}$$

multiply by $\frac{1}{2}$

$$\frac{1}{2}z = \frac{1}{8}-\frac{1}{16}+\frac{1}{32}-\frac{1}{64}+\frac{1}{128}-\frac{1}{256}+\frac{1}{512}$$

$$\frac{3}{2}z = \frac{1}{4}+\frac{1}{512} \Rightarrow z = \frac{1}{6}+\frac{1}{256\times 3} = \frac{1}{6}\left(1+\frac{1}{128}\right) = \frac{1}{6}\left(\frac{129}{128}\right)$$