Name: KEY ID: KEY Sec: 34 23

MATH-102 Term-132 Class-QUIZ-12

1) Expand using partial fractions (DO NOT determine the coefficients) $\int \frac{dx}{x^6-1}$.

$(x^6-1) = (x^3-1)(x^3+1) = (x-1)(x^2+x+1)(x+1)(x^2-x+1)$

$$\frac{1}{x^6-1} = \frac{A}{x-1} + \frac{B}{x+1} + \frac{Cx+D}{x^2+x+1} + \frac{Ex+F}{x^2-x+1}$$

2) Evaluate $\int \frac{3x^2+1}{x^4-1}dx = I$ $x^4-1 = (x^2-1)(x^2+1) = (x-1)(x+1)(x^2+1)$

$$\frac{3x^2+1}{(x^4-1)} = \frac{A}{x-1} + \frac{B}{x+1} + \frac{Cx+D}{x^2+1}$$

multiply by $(x-1)(x+1)(x^2+1)$

$3x^2+1 = A(x+1)(x^2+1) + B(x-1)(x^2+1) + (Cx+D)(x^2-1)$

$x=1$: $4A = 4 \rightarrow A=1$

$x=-1$: $-4B = 4 \rightarrow B=-1$

$x=0$: $1 = A - B - D \rightarrow D=1$

$x=2$: $13 = 15A + 5B + 6C + 3 \rightarrow 10 - 15 + 5 = 6C \rightarrow C=0$

$$I = \int \frac{1}{x-1}dx + \int \frac{-1}{x+1}dx + \int \frac{1}{x^2+1}dx$$

$$= \ln|x-1| - \ln|x+1| + \tan^{-1}x + C$$

$$= \ln\left|\frac{x-1}{x+1}\right| + \tan^{-1}x + C$$

2) Evaluate $\int \frac{\sin x\,dx}{\sqrt{9+\cos^2 x}} = I$ Let $u = \cos x \rightarrow du = -\sin x\,dx$

$I = -\int \frac{du}{\sqrt{9+u^2}}$ Let $u = 3\tan\theta \rightarrow du = 3\sec^2\theta\,d\theta$

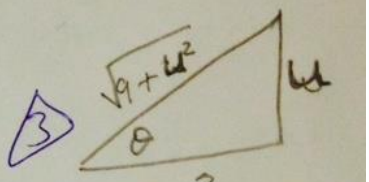
$= -\int \frac{3\sec^2\theta\,d\theta}{3\sqrt{\sec^2\theta}} = -\int \sec\theta\,d\theta = -\ln|\sec\theta + \tan\theta| + \hat{C}$

$= -\ln\left|\frac{\sqrt{9+u^2}}{3} + \frac{u}{3}\right| + \hat{C}$

$= -\ln|\sqrt{9+u^2} + u| + \ln 3 + \hat{C}$

$= -\ln|\sqrt{9+u^2} + u| + C$

$= -\ln|\sqrt{9+\cos^2 x} + \cos x| + C$



$\sec\theta = \frac{\sqrt{9+u^2}}{3}$

4) Determine whether the improper integral is convergent or divergent $\int_2^\infty \frac{\sqrt[3]{x^2+1}}{(\ln x)^4}\,dx$