

1) Evaluate the improper integral

$$\int_0^4 \frac{dx}{\sqrt{4-x}}$$

The function is not defined at  $x=4$

$$\therefore \int_0^4 \frac{dx}{\sqrt{4-x}} = \lim_{t \rightarrow 4^-} \int_0^t \frac{dx}{\sqrt{4-x}}$$

$$\int_0^t \frac{dx}{\sqrt{4-x}} = \int_0^t (4-x)^{-1/2} dx = \left[ \frac{(4-x)^{1/2}}{1/2} \right]_0^t = \left[ -2\sqrt{4-x} \right]_0^t$$

$$= -2(4-t)^{1/2} + 4$$

$$\int_0^4 \frac{dx}{\sqrt{4-x}} = \lim_{t \rightarrow 4^-} \left[ -2(4-t)^{1/2} + 4 \right] = 4$$

2) Determine whether the improper integral is convergent or divergent

$$\int_1^{\infty} \frac{\sqrt{x+1}}{x^2} dx$$

use limit comparison test with  $\int_1^{\infty} \frac{1}{x^{3/2}} dx$

also it is known that  $\int_1^{\infty} \frac{dx}{x^{3/2}}$  is convy ( $p > 1$ )

$$L = \lim_{x \rightarrow \infty} \frac{\frac{\sqrt{x+1}}{x^2}}{\frac{1}{x^{3/2}}} = \lim_{x \rightarrow \infty} \left( \frac{\sqrt{x+1}}{x^2} \cdot \frac{x^{3/2}}{1} \right) = \lim_{x \rightarrow \infty} \sqrt{\frac{x+1}{x}}$$

$= 1 \Rightarrow L = 1$ . Hence both integrals are convy.