Name: KEY | ID: KEY | Sec: 34(11:00-11:50) 23(1:10-2:00)

MATH-102 Term-132 CQ-17 FORM A

(show all your work and circle one letter to get a full mark or you will get zero)

1) The first three nonzero terms of the Taylor series $f(x)=\sqrt{x}$ about $a=1$ are given by

(a) $1-\frac{1}{2}(x-1)+\frac{1}{4}(x-1)^2+\frac{3}{8}(x-1)^3$

(b) $\cancel{1+\frac{1}{2}(x-1)-\frac{1}{4}(x-1)^2+\frac{3}{8}(x-1)^3}$ $1+\frac{1}{2}(x-1)-\frac{1}{8}(x-1)^2+\frac{1}{16}(x-1)^3$

(c) $1+\frac{1}{2}(x-1)-\frac{3}{4}(x-1)^2+\frac{5}{8}(x-1)^3$

(d) $1+\frac{1}{2}(x-1)+(x-1)^2+\frac{3}{8}(x-1)^3$

(e) $1+(x-1)-\frac{1}{4}(x-1)^2+\frac{3}{8}(x-1)^3$

(f) none of the above

$f(x)=x^{1/2}$ $\quad f(1)=1$ $\quad \frac{f(1)}{0!}=1$

$f'(x)=\frac{1}{2}x^{-1/2}$ $\quad f'(1)=\frac{1}{2}$ $\quad \frac{f'(1)}{1!}=\frac{1}{2}$

$f''(x)=-\frac{1}{4}x^{-3/2}$ $\quad f''(1)=-\frac{1}{4}$ $\quad \frac{f''(1)}{2!}=-\frac{1}{8}$

$f'''(x)=\frac{3}{8}x^{-5/2}$ $\quad f'''(1)=\frac{3}{8}$ $\quad \frac{f'''(1)}{3!}=\frac{1}{16}$

$f(x)=1+\frac{1}{2}(x-1)-\frac{1}{8}(x-1)^2+\frac{1}{16}(x-1)^3+\cdots$

2) Using the binomial series, we have, for $|x|<\frac{1}{2}$

$f(x)=\sqrt{4+32x^3}=\sqrt{4}\sqrt{1+8x^3}$

$=2(1+8x^3)^{1/2}$

$k=1/2$

(a) $1+\frac{1}{4}x^3+8x^6+32x^9$

(b) $1+\frac{1}{4}x^3-8x^6+32x^9$

(c) $2+\frac{1}{2}x^3+16x^6+64x^9$

(d) $2-\frac{1}{2}x^3-16x^6-64x^9$

(e) $2+\cancel{\frac{1}{2}}8x^3-16x^6+64x^9$

(f) none of the above

$f(x)=2\left(1+\frac{1}{2}(8x^3)+\frac{(\frac{1}{2})(\frac{1}{2}-1)}{2!}(8x^3)^2+\frac{(\frac{1}{2})(\frac{1}{2}-1)(\frac{1}{2}-2)}{3!}(8x^3)^3+\cdots\right)$

$=2\left(1+4x^3-\frac{1}{8}8^2x^6+\frac{1}{16}8^3x^9+\cdots\right)$

$=2+8x^3-16x^6+64x^9+\cdots$

3) The power series representation for the function $f(x)=\dfrac{9x^2}{2+6x^2}$ is

(a) $\sum_{n=0}^{\infty}(-1)^n3^nx^{2n}$

(b) $\sum_{n=0}^{\infty}(-1)^n3^{n+2}x^{2n+2}$

(c) $\sum_{n=0}^{\infty}\frac{(-1)^n3^{n+2}x^{2n+2}}{2}$

(d) $\sum_{n=0}^{\infty}(-1)^n3^{n+1}x^{2n+2}$

(e) $\sum_{n=0}^{\infty}\frac{(-1)^n3^{n-2}x^{2n+2}}{2}$

(f) none of the above

$\frac{1}{2+6x^2}=\frac{1}{2}\cdot\frac{1}{1+3x^2}$

$\frac{1}{1-x}=\sum_{n=0}^{\infty}x^n$

$x\to -3x^2$

$\frac{1}{1+3x^2}=\sum_{n=0}^{\infty}(-1)^n3^nx^{2n}$

$\frac{1}{2+6x^2}=\sum_{n=0}^{\infty}\frac{(-1)^n3^nx^{2n}}{2}$

$\Rightarrow \frac{9x^2}{2+6x^2}=\sum_{n=0}^{\infty}\frac{(-1)^n3^{n+2}x^{2n+2}}{2}$

4) if the Maclaurin series of $e^x\cos x$ is $A+Bx+Cx^2+Dx^3+\cdots\cdots$ then C+D =

(a) 1/2

(b) 2/3

(c) 1/6

(d) 0

(e) 1

(f) none of the above

$e^x\cos x=\left(1+x+\frac{1}{2}x^2+\frac{1}{6}x^3+\cdots\right)\left(1-\frac{1}{2}x^2+\frac{1}{4!}x^4\cdots\right)$

Coeff of $x^2=C=-\frac{1}{2}+\frac{1}{2}=0$

Coeff of $x^3=D=0-\frac{1}{2}+0+\frac{1}{6}$

$=\frac{-3+1}{6}=-1/3$

5) $\int \frac{1}{x^2}\cos(x^3)dx =$

(a) $\sum_{n=0}^{\infty} \frac{(-1)^n x^{6n-1}}{(2n)!(6n-1)}$

(b) $\sum_{n=0}^{\infty} \frac{(-1)^n x^{6n-2}}{(2n)!(6n-2)}$

(c) $\sum_{n=0}^{\infty} \frac{(-1)^n x^{6n-1}}{(2n+1)!(6n-1)}$

(d) $\sum_{n=3}^{\infty} \frac{(-1)^n x^{6n-1}}{(2n)!(6n-1)}$

(e) $\sum_{n=0}^{\infty} \frac{(-1)^n x^{6n-1}}{(2n)!(3n-1)}$

(f) none of the above

$$\cos x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!}$$

$$\cos x^3 = \sum_{n=0}^{\infty} \frac{(-1)^n x^{6n}}{(2n)!}$$

$$\frac{1}{x^2}\cos x^3 = \sum_{n=0}^{\infty} \frac{(-1)^n x^{6n-2}}{(2n)!}$$

$$\int \frac{1}{x^2}\cos(x^3)dx =$$

$$C + \sum_{n=0}^{\infty} \frac{(-1)^n x^{6n-1}}{(6n-1)(2n)!}$$

6) Let $g(x) = x^3 \tan^{-1} x$ and let $g''(x) = \sum_{n=0}^{\infty} c_n x^n$ then $c_{10} =$

(a) 144/8

(b) 44/3

(c) 14/5

(d) 0

(e) 10

(f) none of the above

$$\tan^{-1} x = x - \frac{1}{3}x^3 + \frac{1}{5}x^5 - \frac{1}{7}x^7 + \frac{1}{9}x^9 + \cdots$$

$$f = x^3 \tan^{-1} x = x^4 - \frac{1}{3}x^6 + \frac{1}{5}x^8 - \frac{1}{7}x^{10} + \frac{1}{9}x^{12} + \cdots$$

$$f' = 4x^3 \cdots\cdots + \frac{1}{9}(12)x^{11} + \cdots$$

$$f'' = 12x^2 + \cdots\cdots + \frac{(11)(12)}{9}x^{10} + \cdots$$

$$\text{Coeff of } x^{10} = c_{10} = \frac{(11)(\cancel{12})^4}{\cancel{9}_3}$$

$$= 44/3$$

$$f = x^3 \tan^{-1} x = \sum_{n=0}^{\infty} \frac{x^{2n+4}}{(2n+1)}(-1)^n$$

$$f' = \sum_{n=0}^{\infty} \frac{(2n+4)x^{2n+3}}{(2n+1)}(-1)^n$$

$$f'' = \sum_{n=0}^{\infty} \frac{(2n+4)(2n+3)}{(2n+1)} x^{2n+2} (-1)^n$$

$$+ \frac{(12)(11)}{9}$$

$n = 4$

7) $(e-2) - \frac{(e-2)^2}{2} + \frac{(e-2)^3}{3} - \frac{(e-2)^4}{4} + \frac{(e-2)^5}{5} - \cdots\cdots =$

(a) $\ln(e-1)$

(b) $\ln(1-e)$

(c) $\ln(e-2)$

(d) $\ln(2-e)$

(e) $2\ln(e-1)$

(f) none of the above

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \frac{x^5}{5} + \cdots$$

let $x = e-2$

$$\ln(e-1) = (e-2) - \frac{(e-2)^2}{2} + \frac{(e-2)^3}{3} - \frac{(e-2)^4}{4} + \cdots$$