MATH 102.6 (Term 132)

Quiz 5 (Sects. 10.1, 10.2 & 10.3) Duration: 20mn

Name: ID number:

1.)(3pts)Find the limit of the sequence $\{\frac{n}{3}\cos\frac{3}{n}-\frac{n}{3}\}_{n=1}^{\infty}$.

2.)(3pts)What is the value of sum $\sum_{n=1}^{\infty}\frac{(-2e)^{1-3n}}{(\sqrt{3})^{2-3n}}$.

3.)(4pts)Do the following series converge or diverge $\sum_{n=1}^{\infty}\frac{3n}{(n^2+1)^2}$, $\sum_{n=2}^{\infty}\frac{1}{n(1+(\ln n)^2)}$ (Hint: use the integral test for both series).

1) $\frac{n}{3}\cos\frac{3}{n}-\frac{n}{3}=\frac{n}{3}\left(\cos\frac{3}{n}-1\right)$

$=\dfrac{\cos\frac{3}{n}-1}{\frac{3}{n}}$

Let $\frac{3}{n}=x$

when $n\to\infty$, $x\to 0$

$\lim_{x\to 0}\dfrac{\cos x-1}{x}=\lim_{x\to 0}\dfrac{-\sin x}{1}=0$

(Hopital rule)

$\Rightarrow\boxed{\lim_{n\to\infty}\left[\frac{n}{3}\cos\frac{3}{n}-\frac{n}{3}\right]=0}$

2.) $\sum_{n=1}^{\infty}\frac{(-2e)}{(\sqrt{3})^2}\left(\frac{-2e}{\sqrt{3}}\right)^{-3n}$

$=-\frac{2e}{3}\sum_{n=1}^{\infty}\left(\left(\frac{\sqrt{3}}{-2e}\right)^3\right)^n$

$=-\frac{2e}{3}\left(-1+\sum_{n=0}^{\infty}\left(-\frac{3\sqrt{3}}{8e^3}\right)^n\right)$

$\left|\frac{3\sqrt{3}}{8e^3}\right|<1$

$=-\frac{2e}{3}\left(-1+\frac{1}{1+\frac{3\sqrt{3}}{8e^3}}\right)$

$=+\dfrac{2e\sqrt{3}}{8e^3+3\sqrt{3}}$

3) $\int_1^{\infty}\frac{3x}{(x^2+1)^2}dx=\left[-\frac{3}{2}\frac{1}{x^2+1}\right]_1^{\infty}=\frac{3}{4}$

$\Rightarrow\sum_{n=1}^{\infty}\frac{3n}{(n^2+1)^2}$ Converges

$\int_2^{\infty}\frac{dx}{x(1+(\ln x)^2)}=\int_{\ln 2}^{\infty}\frac{du}{1+u^2}=\left[\tan^{-1}u\right]_{\ln 2}^{\infty}$

$=\frac{\pi}{2}-\tan^{-1}(\ln 2)$

$\Rightarrow\sum_{n=2}^{\infty}\frac{1}{n(1+(\ln n)^2)}$ Converges.