MATH 102.6 (Term 132)

Quiz 4 (Sects. 8.2, 8.3 & 8.4) Duration: 20mn

Name: ID number:

1.)(4pts)Evaluate the integral $I = \int \frac{1}{x\sqrt{4-x^2}}dx$.

2.)(6pts)Evaluate the integral $J = \int \frac{1}{(x^2-1)(x+3)}dx$.

3.)Write the function $f(x) = \frac{2x^2+1}{x^2(x^2+x+1)^2(x-2)}$ in partial fraction (Do not evaluate the constants).

(2+4)

1) $I = \int \frac{1}{x\sqrt{4-x^2}}\,dx$

Let $x = 2\sin\theta$, $\theta \in]-\frac{\pi}{2}, \frac{\pi}{2}[$

$dx = 2\cos\theta\, d\theta$

$4-x^2 = 4-4\sin^2\theta = 4(\cos^2\theta)$

$\cos\theta > 0 \Rightarrow \sqrt{4-x^2} = 2\cos\theta$

$\Rightarrow I = \int \frac{2\cos\theta\, d\theta}{4\sin\theta\cos\theta} = \int \frac{1}{2}\frac{d\theta}{\sin\theta}$

$= \frac{1}{2}\ln|\csc\theta - \cot\theta| + C$

$\sin\theta = \frac{x}{2} \Rightarrow \csc\theta = \frac{2}{x}$

$\cot\theta = \cos\theta\csc\theta = \frac{2}{x}\sqrt{1-\frac{x^2}{4}}$

$\Rightarrow I = \frac{1}{2}\ln\left|\frac{2}{x} - \frac{2}{x}\sqrt{1-\frac{x^2}{4}}\right| + C$

or $= -\frac{1}{2}\ln\left|\frac{x}{2} + \frac{2}{x}\sqrt{1-\frac{x^2}{4}}\right| + C$

2) $J = \int \frac{dx}{(x^2-1)(x+3)}$

$\frac{1}{(x^2-1)(x+3)} = \frac{a}{x-1} + \frac{b}{x+1} + \frac{c}{x+3}$

$a = \frac{1}{(x+1)(x+3)}\Big|_{x=1} = \frac{1}{8}$

$b = \frac{1}{(x-1)(x+3)}\Big|_{x=-1} = \frac{-1}{4}$

$c = \frac{1}{x^2-1}\Big|_{x=-3} = \frac{1}{8}$

$\Rightarrow J = \int\left(\frac{1/8}{x-1} - \frac{\frac{1}{4}}{x+1} + \frac{\frac{1}{8}}{x+3}\right)dx$

$= \frac{1}{8}\ln|x-1| - \frac{1}{4}\ln|x+1| + \frac{1}{8}\ln|x+3| + C$

3) $\frac{2x^2+1}{x^2(x^2+x+1)(x-2)} = \frac{a}{x} + \frac{b}{x^2} + \frac{cx+d}{x^2+x+1} + \frac{ex+f}{(x^2+x+1)^2} + \frac{g}{x-2}$