MATH 102.6 (Term 132)

Quiz 3 (Sects. 7.1, 7.3 & 8.1) Duration: 20mn

Name: ID number:

1.)(4pts)Find all real numbers x such that $\cosh 2x = 2$.

2.)(6pts)Evaluate $I = \int_1^e x^4(\ln x)^2 dx$.

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1) $\cosh 2x = 2 \iff \frac{e^{2x}+e^{-2x}}{2} = 2$

$\iff e^{2x} + e^{-2x} - 4 = 0$

$\iff e^{4x} - 4e^{2x} + 1 = 0$

let $u = e^{2x}$

$\Rightarrow u^2 - 4u + 1 = 0$

$\Delta = 16 - 4 = 12$

$u_1 = \frac{4-\sqrt{12}}{2}$, $u_2 = \frac{4+\sqrt{12}}{2}$

Thus, $e^{2x} = 2-\sqrt{3}$

and $\boxed{x = \frac{1}{2}\ln(2-\sqrt{3})}$

$e^{2x} = 2+\sqrt{3}$

and $\boxed{x = \frac{1}{2}\ln(2+\sqrt{3})}$

2) $I = \int_1^e x^4(\ln x)^2 dx$

We integrate by part

$u = (\ln x)^2 \Rightarrow u' = \frac{2}{x}\ln x$

$v' = x^4 \Rightarrow v = \frac{x^5}{5}$

$I = \left[\frac{x^5}{5}(\ln x)^2\right]_1^e - \frac{2}{5}\int_1^e x^4 \ln x\, dx$

We apply the integration by part one more time.

$u = \ln x \Rightarrow u' = \frac{1}{x}$

$v' = x^4 \Rightarrow v = \frac{x^5}{5}$

$\Rightarrow$

$I = \frac{e^5}{5} - \frac{2}{5}\left[\frac{x^5}{5}\ln x\right]_1^e + \frac{2}{5}\int_1^e \frac{x^4}{5} dx$

$= \frac{e^5}{5} - \frac{2}{25}e^5 + \frac{2}{5\cdot 5}\left[\frac{x^5}{5}\right]_1^e$

$= \frac{e^5}{5}\left(1 - \frac{2}{5} + \frac{2}{25}\right) - \frac{2}{125}$

$\boxed{I = \frac{1}{125}(17e^5 - 2)}$