Dept of Mathematics and Statistics
King Fahd University of Petroleum & Minerals

AS482: Actuarial Contingencies II
Dr. Mohammad H. Omar
Final Exam Term 132 FORM A
Saturday May 17 2014
8.00am-10.30am

Name_____ ID#:_____ Serial #:___

Instructions.

1. Please turn off your cell phones and place them under your chair. Any student caught with mobile phones on during the exam will be considered under the **cheating rules** of the University.
2. If you need to leave the room, please do so quietly so not to disturb others taking the test. No two person can leave the room at the same time. No extra time will be provided for the time missed outside the classroom.
3. Only materials provided by the instructor can be present on the table during the exam.
4. Do not spend too much time on any one question. If a question seems too difficult, leave it and go on.
5. Use the blank portions of each page for your work. Extra blank pages can be provided if necessary. If you use an extra page, indicate clearly what problem you are working on.
6. Only answers supported by work will be considered. Unsupported guesses will not be graded.
7. While every attempt is made to avoid defective questions, sometimes they do occur. In the rare event that you believe a question is defective, the instructor cannot give you any guidance beyond these instructions.
8. Mobile calculators, I-pad, or communicable devices are disallowed. Use regular scientific calculators or financail calculators only. Write important steps to arrive at the solution of the following problems.

The test is 150 minutes, GOOD LUCK, and you may begin now!

Question	Total Marks	Marks Obtained	Comments
1	4+4=8		
2	4+4=8		
3	8		
4	3+3=6		
5	3+3=6		
6	3+3=6		
7	4+4=8		
8	1+4=5		
9	1+4=5		
Total	60		

Extra blank page

1. (4+4=8 points) A block of 500 fully discrete insurances, issued at age 70, are in force at age 79. The gross premium is $G = 16$, the ninth gross premium reserve is ${}_9V = 115.00$, the tenth gross premium reserve is ${}_{10}V = 128.83$, the tenth year death benefit is 1000, the tenth year withdrawal benefit is 110, and the assumed interest rate is 0.06. Expenses are 3 per policy, incurred at the beginning of the year, and there are no claim settlement expenses. Withdrawals can occur only at the end of the contract year. The assumed decrement rates are $q_{79}'^{(d)} = 0.01$ and $q_{79}'^{(w)} = 0.10$. During the tenth contract year there are 7 **deaths** and 50 **withdrawals**. On this block of policies, calculate, **in order,**

   (a) the **gain** from **mortality** and

   (b) the **gain** from **withdrawal**.

2. (4+4=8 points) Due to the demise of a distant relative, you will receive \$25 000 in one year that you would like to invest at that the time for two more years. The following table provides n-year forward one-year rates available for investment on zero coupon bonds.

n	0	1	2	3	4
$f_{n,1}$	4%	5%	6%	7%	8%

(a) Using the rates in the table above, what rate can be locked in for the investment period?

(b) What transaction should be entered into today in order to lock in the rate from part (a)? Include the terms and principal amounts of the two transactions.

3. (8 points) Consider a Universal Life (UL) contract with face amount 100 000 issued at age x. The contributions, mortality rates, and withdrawal rates are shown in the Table below for the first five years where it is assumed that withdrawals occur only at the end of the policy year.

Year t	**Contribution**	$q^{(d)}_{[x]+t-1}$	$q^{(w)}_{[x]+t-1}$	$p^{(\tau)}_{[x]+t-1} = \left(1 - q^{(d)}_{[x]+t-1}\right)\left(1 - q^{(w)}_{[x]+t-1}\right)$
1	20 000	0.001	0.02	$(0.999)(0.98) = 0.97902$
2	25 000	0.002	0.02	$(0.998)(0.98) = 0.97804$
3	25 000	0.003	0.03	$(0.997)(0.97) = 0.96709$
4	30 000	0.004	0.04	$(0.996)(0.96) = 0.95616$
5	20 000	0.005	0.05	$(0.995)(0.95) = 0.94525$

Suppose the contract incurs the following expenses:

(i) Commissions of 80% in first year and 5% in renewal years.

(ii) Sales expense of 110% of first year commission.

(iii) Acquisition expense of 50% of first year premium plus 100 per policy.

(iv) Annual maintenance expense of 0.20% of face amount plus 50 per policy.

(v) Claim settlement expense of 100 per policy.

Calculate the actuarial present value of **expenses** at policy issue.

4. (3+3=6 points) An equity-linked UL contract uses the annual **point-to-point indexing** method, with a 70% **current** participation rate, a 50% **guaranteed** participation rate, **no** index **cap**, and a **0% guaranteed** index **floor**. The option cost at issue, as a percent of the policy value to which the index benefit is applied, is 7% for a 100% participation rate. The **valuation interest rate** is 4%. Calculate

(a) the **implied guaranteed interest rate** for the **initial** term, and
(b) the implied guaranteed interest rate for the guarantees **beyond the current term**.

5. (3+3=6 points) A variable annuity contract is purchased on 1/1/07 with a deposit of \$50 000. On 12/31/07 the contract value is \$53 000. The contract holder makes a second deposit of \$10 000 on 6/30/08/. Due to a falling market, the contract value on 12/31/08 has dropped to \$59 000. The contract holder dies on 4/5/09, at which time the contract value is \$61 000. No withdrawals were taken from the contract.

a) What is the guaranteed minimum death benefit (GMDB) under the **return of premium** option?

b) What is the GMDB under the **annual step-up** option?

6. (3+3=6 points) Suppose an 85 year old investor with a certificate of deposit matring for \$210 000 purchases a variable immediate annuuity, choosing the life option with no term certain. The annuity factor is $a_{85} = 8$, based on an AIR of 3%. The unit value at the time of purchases is $PUV_1 = 7.234$. The net investment factors for the next two contract years are $NIF_2 = 1.048$ and $NIF_3 = 1.006$. Calculate the number of annuity payment units and the first, second and third annual annuity payment amounts.

7. (4+4=8 points) Consider a 3-state model with states 1 (*Alive and Healthy*), 2 (*Alive but Disabled*), and 3 (*Not Alive*). Suppose the only forces of transition are $\lambda_{12}(s) = 0.10s + 0.20$, $\lambda_{13}(s) = 0.30$, $\lambda_{21}(s) = 0.50$, and $\lambda_{23}(s) = 0.125s + 0.20$, for $0 \leq s \leq 2$ in all cases. Using the value $\Delta r = 0.5$ and for $r = 0.5$, 1.0, 1.5, 2.00, calculate the approximate values of

   a) ${}_rp_{11}^{(0)}$ and

   b) ${}_rp_{12}^{(0)}$

8. (1+4=5 points) For (x) and (y) with independent future lifetimes, you are given:

(i) (x) is subject to a **uniform** distribution of deaths over each year of age.
(ii) (y) is subject to a **constant** force of mortality of 0.25.
(iii) $q_{\overset{1}{xy}} = 0.125$.
Calculate q_x.

A) 0.130
B) 0.141
C) 0.167
D) 0.214
E) 0.250

Work Shown (4 points):

Hence the answer is ( )

9. (1+4=5 points) The CAS Insurance Company classifies its auto drivers as **Preferred** (State 1) or **Standard** (State 2) at time 0, which is the start of the first year the driver is insured. After issue, drivers are continuously reclassified. For a driver, Anne, you are given:

(i) $[x]$ denotes Anne's age at time 0.
(ii) For $k = 0, 1, 2, \ldots,$

$$p^{11}_{[x]+k} = 0.7 + \frac{0.1}{k+1} \qquad p^{21}_{[x]+k} = 0.4 - \frac{0.2}{k+1}$$

$$p^{12}_{[x]+k} = 0.3 - \frac{0.1}{k+1} \qquad p^{22}_{[x]+k} = 0.6 + \frac{0.2}{k+1}$$

(iii) Anne is classified **Preferred** at the start of year 2.

Calculate the probability that Anne is classified **Preferred** at the start of year 4.
A) 0.55
B) 0.59
C) 0.63
D) 0.67
E) 0.71
Work Shown (4 points):

Hence the answer is ( )

END OF TEST PAPER